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Extending a global sensitivity analysis technique to models with correlated parameters

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Abstract

The identification and representation of uncertainty is recognized as an essential component in model applications. One important approach in the identification of uncertainty is sensitivity analysis. Sensitivity analysis evaluates how the variations in the model output can be apportioned to variations in model parameters. One of the most popular sensitivity analysis techniques is Fourier amplitude sensitivity test (FAST). The main mechanism of FAST is to assign each parameter with a distinct integer frequency (characteristic frequency) through a periodic sampling function. Then, for a specific parameter, the variance contribution can be singled out of the model output by the characteristic frequency based on a Fourier transformation. One limitation of FAST is that it can only be applied for models with independent parameters. However, in many cases, the parameters are correlated with one another. In this study, we propose to extend FAST to models with correlated parameters. The extension is based on the reordering of the independent sample in the traditional FAST. We apply the improved FAST to linear, nonlinear, nonmonotonic and real application models. The results show that the sensitivity indices derived by FAST are in a good agreement with those from the correlation ratio sensitivity method, which is a nonparametric method for models with correlated parameters.

Keywords: Correlation; FAST; Sensitivity analysis; Uncertainty analysis

1. Introduction

The past 30 years have witnessed the advent of numerous computer models in different application areas including physics, chemistry, environmental sciences and ecology. At the beginning, the structures of models were relatively simple and the outputs of models were assumed to be error free. Today more and more complex models have been developed. A consequence of model complexity is that the errors in both model structure and parameter estimation have increased. In the case of the nonlinearity in the model structure, a small error at the initial simulation time may be exponentially exaggerated in the model output. Thus, the identification and representation of uncertainty (or error) is recognized as an essential component in model application (Iman et al., 2002a, b; Helton et al., 2005).

One important approach in the identification of uncertainty is sensitivity analysis. Sensitivity analysis evaluates how the variations in the model output can be apportioned to variations in model parameters (Crosetto and Tarantola, 2001). Nowadays, many sensitivity analysis techniques are available (Helton, 1993; Saltelli et al., 2000). In the case

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of simple mathematical models, Taylor series expansion can be used to approximate the model and the analytical differential sensitivity index can be derived. However, for complex models, it would be difficult to use the Taylor series approximation, and more advanced techniques are needed. Saltelli et al. (2000) classified the sensitivity techniques into two groups: local sensitivity analysis methods and global sensitivity analysis methods. The local sensitivity analysis techniques examine the local response of the output(s) by varying input parameters one at a time by holding other parameters at central values. The global sensitivity techniques examine the global response (averaged over the variation of all the parameters) of model output(s) by exploring a finite (or even an infinite) region. The local sensitivity analysis is easy to implement. However, the sensitivity index derived by local sensitivity analysis is dependent on the central values of the other parameters. Thus, currently more and more studies are using global sensitivity analysis methods instead of local sensitivity analysis. Many global sensitivity analysis techniques are available, such as Fourier amplitude sensitivity test (FAST) (Cukier et al., 1973; Schaibly and Shuler, 1973; Cukier et al., 1975, 1978; Koda et al., 1979; McRae et al., 1982); the fractional factorial design method (Saltelli et al., 1995; Henderson-Sellers and Henderson-Sellers, 1996; Cryer and Havens, 1999); Plackett–Burman technique (Beres and Hawkins, 2001); Morris method (Morris, 1991); sampling-based methods (Helton et al., 2005); Sobol's method (Sobol', 1993); and McKay's method based on an one-way ANOVA (McKay, 1997).

One of the most popular global sensitivity analysis techniques is FAST. FAST is computationally efficient and can be used for nonlinear and nonmonotonic models. Thus, it has been widely applied in sensitivity analysis of different models, such as chemical reaction models (Haaker and Verheijen, 2004); atmospheric models (Kioutsioukis et al., 2004); nuclear waste disposal models (Lu and Mohanty, 2001); soil erosion models (Wang et al., 2001); and hydrological models (Francos et al., 2003).

One limitation of FAST is that it can only be applied for models with independent parameters (Rodriguez-Camino and Avissar, 1998). However, in many cases, the parameters are correlated with one another. For example, in meteorology, the central pressure of the storm is correlated with the radius of the maximum wind (Iman et al., 2002a). In this paper, we propose to extend FAST to models with correlated model parameters.

2. Method

2.1. Review of FAST

The core idea of FAST is to assign each parameter with a distinct integer frequency (characteristic frequency) through a periodic sampling function. Then, for a specific parameter, the variance contribution can be singled out of the model output by the characteristic frequency based on a Fourier transformation.

The following review is based on the discrete domain and the Fourier transformation is conducted over the real values. It is slightly different than the standard method. However, the main theory is the same. For a standard FAST method review, please refer to McRae et al. (1982) and Saltelli et al. (2000).

2.1.1. Fourier transformation and variance decomposition

Let us consider a computer model $Y = f(x_1, x_2, \ldots, x_n)$, where n is the number of independent parameters. Assume that the domain of independent parameters is a hypercube

$$\Omega_n = (X|x_i^{(\mathrm{Min})} < x_i < x_i^{(\mathrm{Max})}; i = 1, \ldots, n), \tag{1}$$

where $x_i^{(\mathrm{Min})}$ and $x_i^{(\mathrm{Max})}$ is the minimum and maximum value for x_i. In FAST, in order to assess the sensitivity of each parameter, a search function is introduced for each parameter to explore the space Ω_n. The search function must let the parameter x_i oscillate periodically and has a characteristic frequency ω_i, which is used to determine the uncertainty contribution in the model output. Various search functions have been proposed (Cukier et al., 1973; Saltelli et al., 1999). Lu and Mohanty (2001) proposed a search function that can be used for parameters of various distributions,

$$x_i = F_i^{-1}\left(\frac{1}{2} + \frac{1}{\pi}\arcsin(\sin(\omega_i s))\right), \quad -\pi \leqslant s \leqslant \pi, \tag{2}$$