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The generalized sequential compound options pricing and sensitivity analysis

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Abstract

This paper proposes a generalized pricing formula and sensitivity analysis for sequential compound options (SCOs). Most compound options described in literatures, initiating by Geske [Geske, R., 1977. The Valuation of Corporate Liabilities as Compound Options. Journal of Finance and Quantitative Analysis, 12, 541–552; Geske, R., 1979. The Valuation of Compound Options. Journal of Financial Economics 7, 63–81.], are simple 2-fold options. Existing research on multi-fold compound options has been limited to sequential compound CALL options whose parameters are constant. The multi-fold sequential compound options proposed in this study are defined as compound options on (compound) options where the call/put property of each fold can be arbitrarily assigned. In addition, the deterministic time-dependent parameters, including interest rate, depression rate and variance of asset price, make the SCOs more flexible. The pricing formula is derived by the risk-neutral method. The partial derivative of a multivariate normal integration, which is an extension of Leibnitz's Rule, is derived in this study and used to derive the SCOs sensitivities. The general results for SCOs presents in this paper can enhance and broaden the use of compound option theory in the study of real options and financial derivatives.

Keywords: Sequential compound option; Project valuation; Real option; Leibnitz's rule; Option pricing; Risk-neutral

JEL classification: G12; G13; G30; C69

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1. Introduction

Compound options, initiating by Geske (1977, 1979), are options with other options as underlying assets. The *fold number* of a compound option counts the number of option layers tacked directly onto underlying options. The original closed form of 2-fold compound option is proposed by Geske (1977, 1979) and constitutes as precedents with respect to later works. Specific multi-fold compound option pricing formulas are proposed by Geske and Johnson (1984a) and Carr (1988) while the pricing formula of sequential compound call (SCC) is proved by Thomassen and Van Wouwe (2001) and Chen (2003). Chen (2002) and Lajeri-Chaherli (2002) simultaneously derive the price formula for 2-fold compound options through the risk-neutral method. Agliardi and Agliardi (2003) generalize the results to 2 fold compound calls with time-dependent parameters, while Agliardi and Agliardi (2005) extend the multi-fold compound calls to parameters varying with time.

Financial applications based on compound option theory are widely employed. Geske and Johnson (1984a) use exotic multi-fold compound options for the American put option, while Carr (1988) presents the pricing formula for sequential exchange options. Corporate debt (Chen, 2003; Geske and Johnson, 1984b) and chooser options (Rubinstein, 1992), as well as *capletions* and *floortions* (options on interest rate options) (Musiela and Rutkowski, 1998) are also priced by compound options.

In addition to the pricing of financial derivatives, compound option theory is widely used in the real option study. This approach originates from Myers (1977) and is followed by Brennan and Schwartz (1985), Pindyck (1988), Trigeorgis (1993, 1996) and so forth. Examples include project valuation of new drugs (Casimon et al., 2004), production and inventory (Cortazar and Schwartz, 1993) and capital budget decision (Duan et al., 2003). Compound option methodology turns out to be very common, and the theory is versatile enough to treat many real-world cases (Copeland and Antikarov, 2003).

However, the sophisticated structure of financial derivatives and their wide deployment in the real options field have revealed the limitations of the current compound option methodology. 2-fold compound options cannot be used as further building blocks to model other financial innovations, but results concerning multi-fold compound options so far have focused only on sequential compound calls. Although Remer et al. (2001, p.97) mention that "... in practice, different project phases often have different risks that warrant different discount rates," the important feature of time-dependent (or fold-dependent) parameters is rarely taken into account by current methodologies.

This paper, using vanilla European options as building blocks, extends the compound option theory to multi-fold sequential compound options (SCOs) with time-dependent parameters as well as alternating puts and calls arbitrarily (see Table 1). An SCO is defined as a (compound) option written on another compound option, where the call/put feature of each fold can be assigned arbitrarily. The SCOs presented in this study also allow deterministic parameters (such as interest rate, depression rate and variance of asset price) to vary over time, hence entitle this paper as a "generalized" SCOs and regard the situation of fold-wise parameters as its special case. This study derives an explicit valuation formula for SCOs by the risk-neutral method, and performs the sensitivity analysis on the result. Compared with the P.D.E. method, more financial intuition is gained by the risk-neutral derivation. Moreover, the partial derivative of a multivariate normal integration (an extension of Leibnitz's rule), is also derived here for the sensitivity analysis.

Multi-fold SCOs with alternating puts and calls and time-dependent parameters can greatly enhance the number of practical applications for compound options, especially in the real option field. Real world cases can often be expressed in terms of options, such as expansion, contraction,