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Sensitivity analysis of shape memory alloy shells

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Abstract

This paper presents procedures for efficient design sensitivity analysis for shape memory alloy (SMA) structures modeled with shell elements. Availability of sensitivity information at low computational cost can dramatically improve the efficiency of the optimization process, as it enables use of efficient gradient-based optimization algorithms. The formulation and computation of design sensitivities of SMA shell structures using the direct differentiation method is considered, in a steady state electro-thermo-mechanical finite element context. Finite difference, semi-analytical and refined semi-analytical sensitivity analysis approaches are considered and compared in terms of efficiency, accuracy and implementation effort, based on a representative finite element model of a miniature SMA gripper.

Keywords: Shape memory alloys; Semi-analytical design sensitivities; Finite differences; Fast reanalysis

1. Introduction

Shape memory alloys (SMAs) are active materials with a high power density, capable of producing comparatively large actuation strains and stresses [1]. Their actuation properties originate from a solid-state phase transformation, which is affected by changes in temperature or stress, and strains associated with this transformation can be used for actuation. SMA actuators are widely used in wire or spring configurations, but upcoming applications in, e.g., medical instrumentation or microsystems also demand more complex shapes. However, designing effective multi-dimensional SMA actuators is a challenging task, due to the complex behavior of the material and the fact that often electrical, thermal and mechanical aspects have to be considered simultaneously. For this reason, interest in the application of systematic computational design approaches, such as design optimization techniques, to the design of SMA structures is increasing.

Design optimization has been applied to SMA wire-based configurations [2,3] and to SMA structures modeled by analytical models [4]. However, the models used in these studies cannot be extended to more general SMA structures. In addition, others have applied heuristic peak stress reduction algorithms [5] to more complex SMA designs [6,7]. However, their approach is less versatile than the more general and systematic design optimization techniques developed in the structural optimization community, based on a formal mathematical problem formulation combined with optimization algorithms (see, e.g., [8,9]). Recently, Dumont and Kuhl [10] have demonstrated design optimization of SMA structures modeled by finite elements, using a genetic algorithm. Genetic algorithms are however known to be rather inefficient, which makes that this optimization approach is limited to relatively small problems.

To realize efficient SMA design optimization suited for a wide range of problems of realistic complexity, the availability of sensitivity information is crucial. Various approaches exist to perform sensitivity analysis, and the available techniques and their characteristics are discussed extensively in dedicated books and review papers [9,11–14]. An essential aspect is that with the appropriate techniques,

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design sensitivities can often be obtained at low computational cost, compared to the response evaluation itself. This advantage is particularly evident in the case of history-independent non-linear models [11]. In that case the analysis itself is quite expensive, since the non-linearity usually requires an incremental–iterative solution strategy. In comparison to this significant computational effort, the sensitivity analysis for path-independent models is far less demanding.

The sensitivity analysis presented in this paper is based on a simple constitutive model for SMA behavior based on the R-phase transformation in NiTi [15]. In contrast to the majority of existing SMA models, this model is history-independent and therefore well suited for use in sensitivity analysis and design optimization. However, the considered model is specifically aimed at the R-phase/austenite transformation in NiTi in a selected temperature range, and is not directly applicable for describing more general SMA behavior. The formulation of SMA constitutive models for general SMA behavior, which include hysteresis and dynamic effects, but which at the same time are sufficiently simple to allow sensitivity analysis and design optimization, still remains a challenge. For a discussion of possible alternatives to the presented approach, such as, e.g., the reduction of complicated models using centre-manifold theory, see Ref. [15] and the references therein. The focus of the present paper is however on the outlined SMA behavior based on the R-phase transformation in particular, which offers attractive properties for actuator applications [1]. For this situation, we develop and evaluate effective sensitivity analysis approaches, which allow gradient-based design optimization.

This paper starts with a brief overview of various sensitivity analysis approaches in Section 2. The present work is aimed particularly at SMA shell structures, as these can generate large actuator displacements through bending deformation. The most general case of actuation by means of resistive heating is considered, which requires a sequentially coupled electrical, thermal and mechanical finite element analysis. Simpler situations, e.g., a given temperature distribution, are also covered by this general formulation. Section 3 discusses the derivation and computation of design sensitivities for SMA shell structures in this setting. Numerical results based on finite difference, semi-analytical and refined semi-analytical sensitivity analysis approaches are subsequently presented and discussed in Section 4, using a representative case study of a miniature SMA gripper, followed by conclusions. The application of the developed sensitivity analysis routines to a gradient-based design optimization procedure of this SMA gripper is outside the scope of the present paper, but is presented in a forthcoming article [16].

2. Sensitivity analysis approaches

Several approaches exist to perform sensitivity analysis, and the relevant techniques and their characteristics are briefly reviewed here. Detailed discussions can be found in dedicated books and reviews [9,11–14].

In the following subsections, the system response of interest will be denoted by f and the state variables by $\boldsymbol{u}$. For simplicity, only a single design variable s is considered, without loss of generality. The response is considered to be a function of both $\boldsymbol{u}$ and s, where the state variables also implicitly depend on the design variable, i.e., $f = f(\boldsymbol{u}(s), s)$. Adjoint formulations are not considered, since for the intended shape optimization problems they are not expected to offer significant advantages over the direct differentiation method.

2.1. Variational approaches

Fig. 1 shows a schematic overview that illustrates how the various approaches to perform sensitivity analysis are related to the governing equations. In so-called *continuum* or *variational* approaches, first the continuum governing equations are differentiated with respect to the design variables, and the resulting sensitivity equations are subsequently discretized. In the discrete approaches, on the other hand, the design differentiation is applied to the discretized governing equations.

In the variational approach, the sensitivity equations are not linked to the discretization used for the analysis. This means that theoretically, a different discretization can be used, which makes this approach more flexible than the other methods. However, in practice, in order to make the method efficient and to limit inconsistencies, it is usually beneficial to use the same discretization. Another advantage is that in case internal routines and quantities of an analysis program can not be accessed, the variational approach could be considered for sensitivity analysis [17]. For the present SMA case, the variational approach is not considered, since other methods are at least equally efficient, and require less implementation effort.

2.2. Discrete approaches: finite differences

Discrete design sensitivity approaches are based on the discretized system equations. A popular sensitivity analysis approach is the use of finite differences to approximate the design derivatives. One or more additional simulations are required to evaluate the responses in perturbed design configurations $f(s \pm \Delta s)$, after completing the nominal analysis $f(s)$, e.g.,

$$\text{Forward finite difference}: \frac{\mathrm{d}f}{\mathrm{d}s} \approx \frac{f(s+\Delta s) - f(s)}{\Delta s}, \qquad (1)$$

$$\text{Central finite difference}: \frac{\mathrm{d}f}{\mathrm{d}s} \approx \frac{f(s+\Delta s) - f(s-\Delta s)}{2\Delta s}. \qquad (2)$$

These equations follow from Taylor approximations of the response at the current design point. Critical for the accuracy of the obtained derivatives is the selection of the proper design perturbation Δs. A larger perturbation worsens truncation errors due to the truncation of the Taylor