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Exact bounds for the sensitivity analysis of structures with uncertain-but-bounded parameters

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Abstract

Based on interval mathematical theory, the interval analysis method for the sensitivity analysis of the structure is advanced in this paper. The interval analysis method deals with the upper and lower bounds on eigenvalues of structures with uncertain-but-bounded (or interval) parameters. The stiffness matrix and the mass matrix of the structure, whose elements have the initial errors, are unknown except for the fact that they belong to given bounded matrix sets. The set of possible matrices can be described by the interval matrix. In terms of structural parameters, the stiffness matrix and the mass matrix take the non-negative decomposition. By means of interval extension, the generalized interval eigenvalue problem of structures with uncertain-but-bounded parameters can be divided into two generalized eigenvalue problems of a pair of real symmetric matrix pair by the real analysis method. Unlike normal sensitivity analysis method, the interval analysis method obtains informations on the response of structures with structural parameters (or design variables) changing and without any partial differential operation. Low computational effort and wide application rang are the characteristic of the proposed method. Two illustrative numerical examples illustrate the efficiency of the interval analysis.

Keywords: Sensitivity; Uncertainty; Interval analysis method; Structural eigenvalue

1. Introduction

The purpose of sensitivity analysis is to work out the structure response or the variety of performance through the transformation of parameters or designing variables [1]:

$$u = u(b_1, b_2, \ldots, b_n), \tag{1}$$

where $b_1, b_2, \ldots, b_n$ are structure parameters or designing variables. Thus, via partial differential operation and bring $b_0 = (b_{10}, b_{20}, \ldots, b_{n0})^{\mathrm{T}}$ into Eq. (1), we have

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$$\left.\frac{\partial u}{\partial b}\right|_{b_0} = \frac{\partial u(b_{10}, b_{20}, \ldots, b_{n0})}{\partial b}. \tag{2}$$

The absolute value of Eq. (2) denotes the sensitivity degree of structure responses or capability to structure parameters.

On condition that $\left.\frac{\partial u}{\partial b}\right|_{b_0} > 0$ the structure response or performance u is monotone increased around the parameter $b_0 = (b_{10}, b_{20}, \ldots, b_{n0})^{\mathrm{T}}$. If $\left.\frac{\partial u}{\partial b}\right|_{b_0} < 0$ the structure response or performance u is monotone degressive around the parameter $b_0 = (b_{10}, b_{20}, \ldots, b_{n0})^{\mathrm{T}}$. Bear in mind Eq. (2), we also gain the transformation of the structure response or performance as

$$\delta u = \frac{\partial u}{\partial b}\delta b. \tag{3}$$

In engineering practice, very often difference operation methods are used instead of differential operation methods, but the results are often unreliable.

However, the above-mentioned normal sensitivity analysis method has many problems:

Case I. The mathematical foundation of the normal sensitivity analysis method is the differential calculus of real analysis. In terms of differential calculus principle, based on partial differential, the sensitivity analysis result is only local information. Namely, in this local bound the structure response or performance is most sensitive to this parameter; but in another local bound, the structure response or performance is likely least sensitive to this parameter. In the same way, the structure response or performance is monotone increased to this parameter in this local bound; but in another local bound, the structure response or performance is likely monotone degressive to this parameter. However, in practice analysis and designing, people were concerned with the sensitivity information of the structure response or performance in a certain large bound. The sensitivity analysis which based on differential calculus cannot satisfy the requirement of such global information. Although we can process normal sensitivity analysis many times and receive the sensitivity information in a certain large bound, the efficiency of the calculation will decrease seriously.

Case II. For most practical engineering, it is impossible to present the parse expressions of structure response or performance in virtue of complexity and legion dimensions. So, usually we use difference or perturbation analysis instead of differential calculus. However, in engineering practice, the variety of parameters or designing variables oversize or undersize will all impact the precision of the sensitivity analysis and present complete incorrect information. As shown in Fig. 1, we have

$$\frac{\delta u_1}{\delta b_1} = \frac{u(b_1) - u(b_0)}{b_1 - b_0} = \frac{u_1 - u_0}{b_1 - b_0} > 0 \tag{4}$$

and

$$\frac{\delta u_2}{\delta b_2} = \frac{u(b_2) - u(b_0)}{b_2 - b_0} = \frac{u_2 - u_0}{b_2 - b_0} < 0. \tag{5}$$

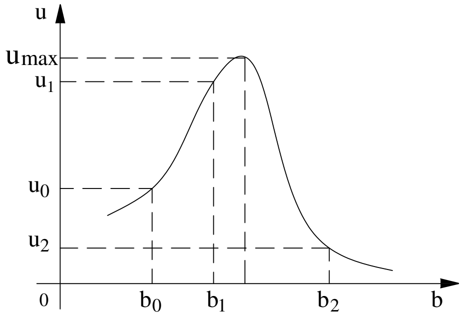

Fig. 1. Distinct nonlinearity function.