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Sensitivity analysis to assess the relative importance of pipes in water distribution networks

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Abstract

Water distribution networks are not randomly constituted systems. On the contrary, their designs are closely related to their hydraulic performance. Recognizing the sundry and relative importance of the different pipes in a water distribution network may help in assessing their impact on the hydraulic performance of the network. This information, in turn, will be helpful in the different aspects of a water distribution network make-up, namely design, planning, control and management. In this paper we propose a methodology to evaluate the relative importance of the pipes in a water distribution network based on the quantification of the uncertainty of the data by means of an error limit analysis performed using sensitivity analysis. By making use of the mathematical model of the network, the estimated steady state together with uncertainty levels, that is to say, a fuzzy estimated steady state of the network can be obtained. We claim that the relative fuzziness of the different flowrates, which can be calculated with inexpensive computational cost, provides a good importance index for the pipes. We show by using an example, the application of this methodology.

Keywords: Water distribution networks; Sensitivity analysis; Error limit analysis; Uncertainty

1. Introduction

Water distribution networks (WDN) are not randomly constituted systems. On the contrary, their designs are closely related to their hydraulic performance. Pipes within any network have different degrees of importance in terms of their impact on the hydraulic performance of the network. Obviously, during operation, the consequences of a breakage of a big pipe close to the source will be different from those of a small peripheral pipe. Also, in the design stage, for example, changing the size of a pipe which is close to a source will have greater impact on the hydraulic performance of the network than making a change to a pipe that is further away. If it is possible to rank pipes in the order of their importance to a particular network, then this information can be exploited in several ways and for different purposes. When the objective is sustainable development, more efficient tools are completely necessary in WDN in order to obtain better Integrated Water Management.

For example, Vairavamoorthy and Ali describe in [19] a method that involves the use of a pipe index vector to control a genetic algorithm search for the optimal design of a water distribution system. The pipe index vector contains

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an index for each pipe being sized and is a measure of the relative importance of that pipe in relation to other pipes. That index is used to exclude sections of the search space that contain impractical or infeasible designs and by doing so enables the genetic algorithm to process healthier populations during its search. The authors claim that this results in the optimal solution being found in a shorter number of generations. The proposed pipe index (PI) is calculated through the solution of a system of N equations, N being the number of demand nodes, with L different right-hand sides, L being the number of lines. It should be noted that the calculation of the PIs involves a heavy computational effort and, in fact, the number of times this is performed must be limited, according to the authors. It goes without saying that this same idea may be used when the design is performed by any of the very popular evolutionary techniques currently available in the literature, ant colony optimization (see [20,14], for example), particle swarm optimization (I. Montalvo, personal communication, May 7th, 2007), harmony search [4], amongst others.

Still within the design stage another interesting application of such a pipe ordering can be devised when reliability is considered. A number of methods have been proposed in the literature to include reliability criteria into the design of a water distribution system. In one of them, [12], Matías, who also uses genetic algorithms as an optimization technique, considers the approach of 'breaking' by turn all the pipes of a specific design to check if all the constraints are fulfilled by the design subjected to this circumstance. If the test is negative the design is suitably penalized. This way, designs belonging to future generations will develop increasing reliability. To undergo those tests, the system must be analyzed for any of those specific 'breakages'. It involves solving the complete set of non-linear equations modelling the water network once for any of the tested pipes. This task will result in a heavy computational burden, especially for large networks. In addition, for a number of pipes the information provided by the test will not be of interest in comparison with the one given by other 'more important' or 'crucial' pipes. Thus, having a measure of the importance of the different pipes will allow skipping over certain pipe tests, which will speed up the optimization process, without influencing relevantly the quality of the solution.

Another example of the use of the relative importance of the pipes can be found in [9]. In this case, it falls into the control and management aspects of a water distribution network. In this paper, a hybrid approach to identify anomalies in water distribution systems is described. The data-driven counterpart consists in a neuro-fuzzy system that, after having been trained, is able to identify fuzzy states of the network corresponding to system anomalies such as leaking pipes, amongst others. The training process depends on a number of parameters, as shown in the paper. The fine-tuning of one of these parameters (the one that controls the fuzziness of a specific state associated to a certain pipe) is performed by taking into account the relative importance of the pipes.

Arulraj and Rao also proposed in [1] an index that can be calculated explicitly as the quotient QL/CD, Q being the flowrate, L the length, C the Hazen–Williams roughness coefficient and D the diameter of a pipe. This index, termed SI, for significance index, was developed for aiding the rehabilitation process of pipe networks. Arulraj and Rao found that pipes needing to be replaced or maintained to improve the pressure in the system are those with a large SI. They defined critical pipes as those pipes that are most sensitive to changes of the values of the diameter D or the Hazen–Williams roughness coefficient C. It appears to be obvious that an index calculated in such a simple way cannot account for the mutual influence of all the variables which are of relative importance to the pipes, at least with a certain level of detail. Thus, it should be considered as a simple but rough way of getting useful information for certain purposes. In fact, Vairavamoorthy and Ali used the SI in [19] as a surrogate measure for their PI only when the genetic algorithm has suitably progressed so that the PI can be replaced with the SI, thus improving the overall computational efficiency of their method.

In this paper we propose a methodology to evaluate the relative importance of the pipes in a water distribution network based on the quantification of the uncertainty of the data by means of error limit analysis performed using sensitivity analysis. First, the mathematical model of the network is used to obtain the steady state and then, given values quantifying the lack of precision for the different magnitudes (demands, piezometric heads etc.), levels of uncertainty for the steady state are obtained by making use of matrix sensitivity, which amounts to inverting a sparse matrix of the same size as that of the original one. This way, a fuzzy estimated steady state of the network is obtained. The relative magnitude of the fuzziness for the flowrate of a pipe reveals its relative importance within the network. This information will be helpful in the different aspects of a water distribution network make-up, namely design, planning, control and management.

The structure of this paper is as follows. First, the mathematical model, already described in [7], is briefly presented. Second, the error limit analysis, which uses the mathematical model, is tackled by an approach based on matrix sensitivity, [2], to obtain levels of uncertainty for the different pipes, which allows the proposed index definition.