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Integral equation and sensitivity analyses of creep behavior for PVDF thin films

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Abstract

This paper presents an analytic study of a linear viscoelasticity constitutive equation involving stress, strain and creep compliance while simultaneously correcting a previously reported investigation [Vinogradov, A.M., Schmidt, V.H., Tuthill, G.F., Bohannan, G.W., (2004). Damping and electromechanical energy losses in the piezoelectric polymer PVDF. Mechanics of Materials 36, 1007–1016]. The constitutive equation is presented as a linear, weakly-singular Volterra integral equation of the second kind in the stress variable. An analytic solution is developed, using the Laplace transform technique, for acquiring the stress history based on a specified creep compliance function and input strain. The time-dependent stress solution is expressed in terms of an infinite series involving the provided strain history. An example is studied involving constant strain input. This example permits an aposteriori error estimate for the stress based on the truncated series. Finally, a novel first-order sensitivity analysis is presented to assist in developing experiments for estimating the parameters associated with the compliance function. Using the proposed first-order sensitivity analysis, it is possible to investigate how the uncertainty associated with these parameters propagate into the stress history.

Keywords: Linear viscoelasticity; Stress; Strain; Compliance; Sensitivity analysis; Volterra equations

1. Introduction

The study of mechanical creep and relaxation of polyvinylidene fluoride (PVDF) has emerged to be of considerable importance for use in microphones, transducer and sensors. This piezoelectric polymer material is commercially available as thin films ranging in thickness from 10 to 760 μm (Vinogradov et al., 2004). Constitutive equation development for quantifying the behavior of such materials is highly important for understanding the underlying physics of these materials. Vinogradov et al. (2004) have suggested that the classical theory of piezoelectricity has significant limitations in terms for representing the electromechanical properties of PVDF's. This brief paper revisits the recent work of Vinogradov et al. (2004) in order to correct numerous typographical and graphical errors that permeate in the paper. Additionally, this paper presents a first-order sensitivity analysis that is highly useful to

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the experimentalist and analyst for (a) designing experiments; and (b) propagating parameter uncertainty onto the stress history.

2. Constitutive relation

Wineman and Rajagopal (2000, p. 65) provide several creep relationships between stress and strain involving the creep compliance kernel. Vinogradov et al. (2004) model the creep behavior of PVDF thin films using the creep relationship given by

$$\epsilon(t) = \sigma(0)C_n(t) + \int_{u=0}^{t} C_n(t-u)\frac{d\sigma}{du}\,du, \quad t \geqslant 0, \ n = 1, 2, \tag{1}$$

where $\sigma(t)$ and $\epsilon(t)$ are stress and strain, respectively as functions of the temporal variable, t. Here, $C_n(t)$ is the creep compliance function under two respective material directions in the PVDF, $n = 1, 2$. Vinogradov et al. (2004) model the creep compliance function as a power law given by

$$C_n(t) = a_n + b_n t^{\alpha_n}, \quad n = 1, 2, \tag{2}$$

from which the parameters $a_n, b_n, \alpha_n,\ n = 1, 2$ are obtained through a least-squares approach. It should be noted that Eqs. (2)–(7) in Vinogradov et al. (2004) contain errors and Figs. 1–4 in Vinogradov et al. (2004) are inconsistent with the data presented in their paper.

The working form or reduction of Eq. (1) is obtained with the aid of integration by parts. This leads to

$$\epsilon(t) = C_n(0)\sigma(t) - \int_{u=0}^{t} \frac{dC_n}{du}(t-u)\sigma(u)\,du, \quad t \geqslant 0, \ n = 1, 2. \tag{3}$$

The power law creep compliance function given in Eq. (2) is next substituted into Eq. (1) to obtain

$$\epsilon(t) = a_n\sigma(t) + b_n\alpha_n \int_{u=0}^{t} \frac{\sigma(u)}{(t-u)^{1-\alpha_n}}\,du, \quad t \geqslant 0, \ n = 1, 2. \tag{4}$$

Technically, Eq. (4) is a linear, Volterra integral equation of the second kind having a weakly-singular kernel (Linz, 1985) when $0 < \alpha_n < 1,\ n = 1, 2$. In some texts, this is considered a generalized Abel integral equation. Linear Volterra integral equations with such kernels are solvable by many methods.

3. General solution to integral equation by laplace transforms

To begin, the solutions provided by Vinogradov et al. (2004, Eqs. (6), (7)) require correction. In fact, Vinogradov et al. (2004) reference Kanwal (1971) (probably page 206) as the source for their solution. Unfortunately, Kanwal (1971) also contains a typographical error. However, the second edition (Kanwal, 1997, p. 229) provides the corrected solution.

To begin, we develop the solution based on the Laplace transform method (Churchill, 1972), where the Laplace transform is defined as

$$\hat{f}(s) = L\{f(t)\} = \int_{t=0}^{\infty} f(t)e^{-st}\,dt, \tag{5}$$

where L is the Laplace transform operator. For this simple integral expression, the only other tool necessary for the analysis will involve the convolution of two functions, given by

$$L^{-1}\{\hat{f}(s)\hat{g}(s)\} = \int_{u=0}^{t} F(u)G(t-u)\,du = F * G, \tag{6}$$

where $F(t) = L^{-1}\{\hat{f}(s)\}$ and $G(t) = L^{-1}\{\hat{g}(s)\}$. For convenience, let us rewrite Eq. (4) as

$$\sigma(t) = \gamma_n\epsilon(t) - \lambda_n \int_{u=0}^{t} \frac{\sigma(u)}{(t-u)^{\beta_n}}\,du, \quad t \geqslant 0, \ n = 1, 2, \tag{7}$$

where $\gamma_n = a_n^{-1}$, $\lambda_n = b_n\alpha_n/a_n$ and $\beta_n = 1 - \alpha_n$. Taking the Laplace transform of Eq. (7) yields

$$\hat{\sigma}(s) = \gamma_n\hat{\epsilon}(s) - \lambda_n L\left\{\int_{u=0}^{t} \frac{\sigma(u)}{(t-u)^{\beta_n}}\,du\right\}, \tag{8a}$$

or upon invoking the convolution theorem displayed in Eq. (6), we obtain

$$\hat{\sigma}(s) = \gamma_n\hat{\epsilon}(s) - \lambda_n L\{\sigma(t)\}L\left\{\frac{1}{t^{\beta_n}}\right\}, \tag{8b}$$

or finally

$$\hat{\sigma}(s) = \gamma_n\hat{\epsilon}(s) - \lambda_n\hat{\sigma}(s)\frac{\Gamma(\alpha_n)}{s^{\alpha_n}}, \tag{8c}$$

where $\Gamma(z)$ is the complete Gamma function with real argument z (Abramowitz and Stegun, 1972) with $L\{t^{\alpha_n - 1}\} = \Gamma(\alpha_n)/s^{\alpha_n}$ (Churchill, 1972, p. 459). Thus, we obtain the transformed stress in terms of the transformed strain as

$$\hat{\sigma}(s) = \frac{\gamma_n\hat{\epsilon}(s)}{1 + \frac{\lambda_n\Gamma(\alpha_n)}{s^{\alpha_n}}}. \tag{9}$$