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# Sensitivity analysis of machine repair problems in manufacturing systems with service interruptions

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**Abstract**

This paper models a manufacturing system consisting of *M* operating machines and *S* spare machines under the supervision of a group of technicians in a repair facility. Machines fail according to a Poisson process, and the repair (service) process of a failed machine may require more than one phase. In each phase, service times are assumed to be exponentially distributed but may be interrupted when the repair facility encounters unpredictable breakdowns. Two models of manufacturing systems are considered. In the first model, technicians repair failed machines at different rates in each phase. In the second model, a two-phase service system with differing numbers of technicians is considered. Profit functions are developed for both models and optimized by a suitable allocation of the number of machines, spares, and technicians in the system. Finally, a sensitivity analysis (see Cao [X.R. Cao, Realization Probabilities: The Dynamics of Queuing Systems, Springer-Verlag: London, 1994; X.R. Cao, The relations among potentials, perturbation analysis, and Markov decision processes, Discrete Event Dynam. Syst.: Theory Applicat. 8 (1998) 71–87]) is performed to provide an approach that quantifies the impact of changes in the parameters on the profit models.


*Keywords:* *k*-Phase service; QR factorization; Optimization; Sensitivity analysis; Service interruptions

## 1. Introduction

This paper analyzes a manufacturing system with *M* operating machines and *S* spare machines producing a single type of product. All machines have the same production rate when operational. Machine breakdowns occur randomly, and there is a group of technicians who services failed machines. As is usually the case, the service process may require a sequence of various repair phases, such as detection of failure factors, minimal repair, and complete repair (see [1]). In addition, the service time for a failed machine may be subject to interruptions for example, breakdowns of the repair facility or absence of technicians.

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Queuing models are effective methods for performance analysis of manufacturing systems (see [2–5]). Sztrik and Bunday [6] made use of the $M/M/1$ queuing model to deal with machine repair problems for a system with $M$ machines and a single technician in which the operating times and the service times were assumed to be exponentially distributed. The model was applied to solve a textile winding problem. Desruelle and Steudel [7] applied the queueing network model to study a machine interference problem in a single-technician manufacturing workcell environment. Recently, Jain et al. [8] used the $M/M/1$ queueing system to examine the reliability characteristics of a machine repair model with $M$ operating machines and $S$ warm spares, where the failed machines may renege when the technician is busy. Jain et al. [9] employed the $M/M/2$ system to study a bi-level switch-over policy of the machine repair model with spares, where two technicians perform a setup before providing services and take vacations when no failed machines are queued for services.

In machine repair models, existing research mostly focuses on service in a single stage encompassing the phases mentioned above. Each service process requires a sequence of constant repair phases, as assumed in Wang [10], Wang and Kuo [11], Ching [12], and others. Wang [10], Wang and Kuo [11] studied the $M/E_k/1$ model in which Erlangian service times are assumed for $k$ stages (phases) of service. This system was applied to model an unloader system in which trains bring coal from various mines. Ching [12] studied Wang's model with different service rates for each phase depending on the number of technicians (see also Buzacott and Shanthikumar [13]). In this paper we consider a more general situation: instead of continuous service we consider possible interruptions of service. Moreover, the service process is generalized to allow for different sequences of service phases each time a machine fails. A detailed description of the manufacturing systems with the two kinds of repair processes considered is given as follows:

### 1.1. Assumptions of Model 1 (k-phase machine repair model)

1. Each of the operating machines fails independently of the state of the others with failure rate $\lambda$. When a machine fails, it is immediately replaced by a spare if one is available. We also assume that spares fail independently of the state of all others with the failure rate $\theta$ ($0 \leqslant \theta \leqslant \lambda$) and that when a spare moves into an operating state, its failure characteristics become those of an operating machine.
2. When an operating machine or a spare fails, it is immediately queued for service by a group of $R$ technicians in a repair facility, that is, failed machines are repaired in order of their breakdowns. Due to failure factors, the service process of a failed machine may need more than one phase. In each phase, the service time is exponentially distributed with a phase-specific rate. At any time, there is at most one failed machine in the service process. Let us assume $\mu_\eta$ is the service rate for the $\eta$th phase, where $\eta = 1, 2, \ldots, k$. All phases of service are mutually independent.
3. A failed machine first enters phase 1 of the service before progressing to phase 2 with probability $\delta$ or completes service with probability $1 - \delta$. If the failed machine enters phase 2 of the service, it either progresses to phase 3 with probability $\delta$ or completes service with probability $1 - \delta$. The repair process continues in this manner for at most $k$ phases. If the failed machine enters phase $k$ of the service, it must complete the service; this case is referred to as complete repair (overhauls).
4. Due to environmental or human factors, any service phase may be interrupted following a Poisson process with rate $\alpha$. The recovery times of service interruptions follow an exponential distribution with mean $1/\beta$, and the service resumes as soon as the interruption ends.

### 1.2. Assumptions of Model 2 (2-phase tandem machine repair model)

Assumptions 1, 2, and 4 in *Model 1* remain the same. However the third assumption is modified in that when a machine fails, it is subject to only a two-phase service process with service times for phases 1 and 2 exponentially distributed with means $1/\mu_1$ and $1/\mu_2$, respectively. Moreover, there are $R_1$ technicians in phase 1 and $R_2$ in phase 2, where $R_1 \geqslant 1$, $R_2 \geqslant 1$, and $R = R_1 + R_2$. Each technician can service only one failed machine in each phase.

Note that the service process of our model varies from Wang's [10] and that it consists of a random sequence of $k$ service phases with a different number of technicians. This sequence is not a fixed $k$ service phase