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Parameter sensitivity analysis for a Drücker–Prager model following from numerical simulations of indentation tests

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ABSTRACT

A parameter sensitivity analysis is carried out from numerical simulations of indention tests. The indented material obeys a simple Drücker–Prager behavior with no hardening rule, involving four material properties. For each of the parameters, the sensitivity is defined as the variation of the error function around a reference indentation curve. It is computed for three loading paths and five indenter shapes: spherical, conical, cylindrical, tetrahedral, and pyramidal. Finally, each of the sensitivities are compared with each other and commented on.

1. Introduction

Indentation tests are commonly used to estimate mechanical properties of materials. The special feature of this experimental method is the relative ease of specimen preparation of the studied material. From a modeling viewpoint, dimensional analysis is commonly used to give the general form of the F—δ (force vs. depth) curve, see e.g. [1,2]. To apprehend the complete formulation of $F(\delta)$, analytic solutions of the displacement field in elasticity and elastoplasticity with power-law hardening has been proposed in [3,4], and it has been examined in [5,6] for linear viscoelasticity. Furthermore, many authors use numerical analysis to link the elastoplastic properties of materials with measurable parameters on the F—δ curve, like the slope S at the beginning of the unloading or the total reversible work W_r, see [7–18] for power hardening rules, [19] for linear hardening rule, and [20–22] for gradient plasticity. Other approaches aim to completely solve the indentation problem, i.e. without using dimensional analysis. In this spirit, numerical inverse analysis has been examined in the case of elastoplastic materials by means of several minimization techniques like neural network, [23,24], simplex method, [25], Kalman filter, [26,27], or adjoint state method [28].

Whereas the above approaches aim to determine material properties, a few of them deal with parameters sensitivities. Bocciarelli et al. [29,30], and Bolzon et al. [31], studied the sensitivity of the imprint residual vertical displacement with respect to the pre-existing principal stresses inside the material. They also showed that the five parameters of an elastoplastic model with kinematic non-linear hardening can be identified if the imprint geometry is considered, whereas it cannot be done if only indentation curves are used. As a general rule, the computation of parameters sensitivities is of great importance since it can indicate the relevance of the parameters estimation. Indeed, for a set of parameters p_i appearing in the material constitutive equations, it is well established that if one parameter p is much less sensitive than others, the condition number of the approximate Hessian matrix $\boldsymbol{J}^T \cdot \boldsymbol{J}$ may be high (with $\boldsymbol{J}$ the gradient of the error function with respect to the parameters), resulting in a divergence of the iterative algorithm used to minimize the error function (e.g. Levenberg–Marquardt). In this case, one has to set a value to p, and proceed to the estimation of other properties. So, for a given set of experimental data, some parameters could be precisely identified if their sensitivity is greater than the sensitivity of the other ones.

This paper deals with a numerical parameters sensitivities study of a material submitted to indentation tests. The sensitivities of the medium properties shall be computed in the case of a Drücker–Prager elastoplastic model. For the sake of completeness, five shapes of indenters are considered (see Fig. 1 for a schematic representation):

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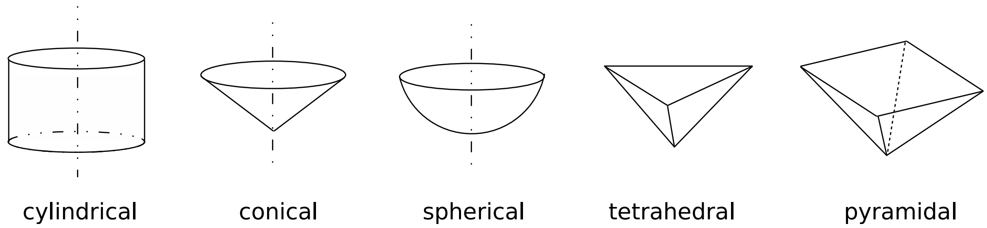

Fig. 1. Different shapes of indenters.

- a spherical indenter (also called *Brinell* indenter),
- a conical indenter,
- a cylindrical indenter,
- a tetrahedral indenter, also known as a *Berkovitch* indenter,
- and finally, a pyramidal indenter, also called *Vickers* indenter.

Indeed, for each of the indenter shapes, three loading–unloading paths are considered. The aim of the paper is to understand the influence of the loading path on parameters sensitivities, and to discuss the identification of a simple Drücker–Prager model properties. Let us now begin with the presentation of the constitutive equations.

2. Rheological model

2.1. General equations

The rheological model considered here is a Drücker–Prager model with no hardening. The total and plastic strains are denoted ε and ε^p, respectively. The Cauchy stress tensor $\boldsymbol{\sigma}$ can then be related to ε and ε^p by

$$\mathrm{d}\boldsymbol{\sigma} = \mathbb{C} : (\mathrm{d}\varepsilon - \mathrm{d}\varepsilon^p), \tag{1}$$

$$\mathrm{d}\varepsilon^p = \mathrm{d}\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}}, \tag{2}$$

with $\mathbb{C}$ the fourth-order tensor of linear isotropic elasticity:

$$\mathbb{C} = \frac{\nu E}{(1-2\nu)(1+\nu)} \mathbb{I} \otimes \mathbb{I} + \frac{E}{2(1+\nu)} \mathbb{J}, \tag{3}$$

$\mathbb{I}_{ij} = \delta_{ij}$, $\mathbb{J}_{ijkl} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$, λ the plastic multiplier, and E and ν the Young's modulus and the Poisson's ratio respectively. The yield function f appearing in (2) is linear with the deviatoric stress q and the first invariant I_1 of $\boldsymbol{\sigma}$:

$$f(\boldsymbol{\sigma}) = q + \alpha I_1 - \sigma^y, \tag{4}$$

taking into account the following definitions:

$$q = \sqrt{\frac{3}{2}\boldsymbol{s} : \boldsymbol{s}}, \quad \boldsymbol{s} = \boldsymbol{\sigma} - p\mathbb{I} \text{ (deviatoric stress tensor)}, \quad I_1 = Tr(\boldsymbol{\sigma}). \tag{5}$$

The following constant properties:

$$\{E, \nu, \alpha, \sigma^y\} \tag{6}$$

are the parameters of the model.

2.2. Values of parameters

To apprehend the parameters sensitivities for different values of (6), three materials with different "resistances" has been considered: claystone, dolomite, and concrete. We are only interested here with typical values of material properties rather than "exact" values of given rocks. It is the reason why more sophisticated Drücker–Prager models found in the literature have been adapted here to obtain numerical values for (6). At first, a complete model for claystone (including damage) has been proposed in [32]; the author's model has been simplified assuming that the friction angle does not depend on the plastic distortion and is equal to the mean value of its two extremal values. Typical values for the dolomite properties have been found in [33,34], except for α that we assumed to be 0.3, a rounded value of the one computed from the friction angle ϕ given by the authors. The parameters of concrete have been computed from [35], by identifying the yield function (4) with the one proposed by the authors. In short, the whole set of parameters (6) associated with the three materials is presented in Table 1. Some considerations about numerical simulations are now presented in the next section.

Table 1
Parameters of the present model for three different rock-like materials

	E (Mpa)	ν	α	σ^y (Mpa)
Claystone	8400	0.14	0.13	8.0
Dolomite	16,200	0.26	0.3	12.4
Concrete	23,700	0.20	0.083	8.3

3. Numerical simulations

3.1. Geometries of indenters

Realistic dimensions have been assigned to the indenters. Namely, the sphere radius of the Brinell indenter and the cylinder radius of the flat indenter are taken to be equal to 1 mm. For sharp indenters, an angle of 68° has been adopted for the tetrahedral and pyramidal indenters, while an angle of 60° has been assigned to the conical indenter.

3.2. Assumptions and meshes

Numerical simulations for the five considered indenter shapes have been carried out with the finite elements software *Code_Aster*®. No friction between bodies is considered. For axisymmetrical indenters, a two-dimensional mesh consisting of axisymmetrical elements has been used while three dimensional meshes have been adopted for the tetrahedral and pyramidal indenters. In all simulations, a master-slave algorithm interacting with the Newton–Raphson iterative method allows the contact problem to be solved, see [36].

As concerns the type of elements, mesh distortion has been observed in the vicinity of the indenter for triangular elements. This distortion involves numerical instabilities of the Newton–Raphson method due to the ill-conditioned stiffness matrix of the indented material. Consequently, four-noded elements (Q4) have been chosen in the vicinity of each axisymmetrical indenter. Furthermore, three-noded elements (T3) have been used elsewhere to ensure a progressively large element size as far from the indenter and the continuity of the displacement field as possible. An illustration of such an hybrid mesh in the case of the flat (cylindrical) indenter is given in Fig. 2, and the resulting force–displacement curve for