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Moment Method Based on Fuzzy Reliability Sensitivity Analysis for a Degradable Structural System

Song Jun, Lu Zhenzhou*

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China

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Abstract

For a degradable structural system with fuzzy failure region, a moment method based on fuzzy reliability sensitivity algorithm is presented. According to the value assignment of performance function, the integral region for calculating the fuzzy failure probability is first split into a series of subregions in which the membership function values of the performance function within the fuzzy failure region can be approximated by a set of constants. The fuzzy failure probability is then transformed into a sum of products of the random failure probabilities and the approximate constants of the membership function in the subregions. Furthermore, the fuzzy reliability sensitivity analysis is transformed into a series of random reliability sensitivity analysis, and the random reliability sensitivity can be obtained by the constructed moment method. The primary advantages of the presented method include higher efficiency for implicit performance function with low and medium dimensionality and wide applicability to multiple failure modes and nonnormal basic random variables. The limitation is that the required computation effort grows exponentially with the increase of dimensionality of the basic random variable; hence, it is not suitable for high dimensionality problem. Compared with the available methods, the presented one is pretty competitive in the case that the dimensionality is lower than 10. The presented examples are used to verify the advantages and indicate the limitations.

Keywords: moment method; fuzziness; sensitivity analysis; fuzzy failure probability

1 Introduction

For most of the failures with gradual performance degradation of the degradable structural system, it is reasonable to describe the failure region with fuzziness[1-2]. The reliability taking the fuzziness of the failure region into consideration is known as the fuzzy reliability. This article focuses on the fuzzy reliability sensitivity, an important output of fuzzy reliability analysis. Similarly, as for the random reliability sensitivity, the fuzzy reliability sensitivity is defined as the partial derivative of the fuzzy failure probability with respect to the distribution parameter of the basic random variable, and it represents the effect of the distribution parameter on the fuzzy failure probability. The normalized fuzzy reliability sensitivity can provide the information of importance ranking for the gradient based on reliability optimization. In addition, the fuzzy reliability sensitivity involves the partial derivative of the fuzzy failure probability with respect to the parameters of the membership function because the fuzzy failure probability is affected by the parameters of the membership function. Currently, there are many well established random reliability sensitivity analysis methods[3-14], e.g. the approxi-

*Corresponding author. Tel.: +86-29-88460480.
E-mail address: zhenzhoulu@nwpu.edu.cn
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mately analytical method for the random reliability sensitivity based on the first order and second moment (FOSM)[3-6], the numerical algorithm based on the original/improved Monte Carlo simulations[7-12], and so on. However, it appears that not many researches have been carried out for the computation of the fuzzy reliability sensitivity. In this article, an approximate transformation from the fuzzy failure probability to the random failure probability is adopted and then the fuzzy reliability sensitivity analysis can be approximately completed by borrowing the random reliability sensitivity method. Because the FOSM based analysis method for random reliability sensitivity is only applicable to the linear performance function with dependent normal variables and the efficiency of numerical algorithms based on the original/improved Monte Carlo simulations need to be improved, such methods for the random reliability sensitivity analysis are not borrowed to complete the fuzzy reliability sensitivity. The moment method based on random reliability sensitivity analysis method, which is constructed by the authors, is used to construct the moment based on fuzzy reliability sensitivity. The advantages of the moment method based on random reliability sensitivity analysis, such as higher efficiency for low and medium dimensionality, wide applicability to the nonnormal random variables, and multiple failure modes, are propagated to the moment method based on fuzzy reliability sensitivity analysis. After the basic concept and the implementation are detailed for the moment method based on fuzzy reliability sensitivity analysis, the illustrations are presented to verify the precision and the efficiency of the presented method, and the limitations are pointed out simultaneously.

2 Basic Concept and Implementation of Fuzzy Reliability Sensitivity Analysis

The presented method for analyzing fuzzy reliability sensitivity includes three steps. First, the definition of fuzzy reliability sensitivity is provided, and its numerical simulation solution is proposed. Because the numerical simulation solution converges to the real value according to the law of large numbers, it is used as a standard solution to verify the presented new method in the illustrations. Second, according to the taken values of the performance function, the integral region for calculating the fuzzy failure probability is split into a series of subregions. The relationship between the fuzzy reliability sensitivity and the random reliability sensitivities within the subregions is constructed. Third, the moment method is established to analyze the random reliability sensitivities of the subregions, and furthermore, the fuzzy reliability sensitivity related to these random reliability sensitivities of subregions is obtained.

2.1 Definition of fuzzy reliability sensitivity

Denoting $\tilde{F}$ as the fuzzy failure region of the researched problem, the fuzziness of $\tilde{F}$ can be described by the membership function $\mu_{\tilde{F}}(g)$ of performance function $g(\boldsymbol{x})$ (where $\boldsymbol{x}$ is the n-dimension basic random vector) to $\tilde{F}$. In the illustrations of this article, three membership functions, i.e. linear form, normal form, and Cauchy form, regularly used in engineering problem are selected to verify the feasibility and rationality of the presented method. The formulae of the three membership functions are shown in Eqs.(1)-(3). For other membership functions, the corresponding extension is straight for ware.

$$\mu_{\tilde{F}}\left[g(\boldsymbol{x})\right]=\begin{cases}1 & g(\boldsymbol{x})\le a_1\\ \dfrac{a_2-g(\boldsymbol{x})}{a_2-a_1} & a_1<g(\boldsymbol{x})<a_2\\ 0 & g(\boldsymbol{x})\ge a_2\end{cases}\tag{1}$$

$$\mu_{\tilde{F}}\left[g(\boldsymbol{x})\right]=\begin{cases}1 & g(\boldsymbol{x})\le b_1\\ \exp\left[-\left(\dfrac{g(\boldsymbol{x})-b_1}{b_2}\right)^2\right] & g(\boldsymbol{x})>b_1\end{cases}\tag{2}$$

$$\mu_{\tilde{F}}\left[g(\boldsymbol{x})\right]=\begin{cases}1 & g(\boldsymbol{x})\le c_1\\ \dfrac{c_2}{c_2+10\left[g(\boldsymbol{x})-c_1\right]^2} & g(\boldsymbol{x})>c_1\end{cases}\tag{3}$$

where a_1, a_2, b_1, b_2, c_1, and c_2 are the parameters of the corresponding membership functions, respectively, and they can be determined by the statistics of the expert experience according to the specific