Available online at www.sciencedirect.com

ScienceDirect

Applied Mathematical Modelling 32 (2008) 2526–2537

APPLIED MATHEMATICAL MODELLING

www.elsevier.com/locate/apm

Skew ray tracing and sensitivity analysis of ellipsoidal optical boundary surfaces

Psang Dain Lin *, Chuang-Yu Tsai

National Cheng Kung University, Department of Mechanical Engineering, Tainan 70101, Taiwan

Received 1 August 2005; received in revised form 1 July 2006; accepted 17 September 2007
Available online 1 October 2007

Abstract

The 4×4 homogeneous transformation matrix is one of the most commonly applied mathematical tools in the fields of robotics, mechanisms and computer graphics. Here we extend further this mathematical tool to geometrical optics by addressing the following two topics: (1) skew ray tracing to determine the paths of reflected/refracted skew rays crossing ellipsoidal boundary surfaces; and (2) sensitivity analysis to determine via direct mathematical analysis the differential changes of the incident point and the reflected/refracted vector with respect to changes in the incident light source. The proposed ray tracing and sensitivity analysis are projected as the nucleus of other geometrical optical computations.

Keywords: Matrix; Sensitivity analysis; Geometric optics

---

1. Introduction

Optical system design requires the accurate determination of the paths of the light rays as they interact with reflective and refractive system components. This can be achieved via the successive application of Snell's well-known laws of reflection and refraction. This extensively employed method is known as ray tracing. The generalized version of this method is referred to as skew ray tracing. Although somewhat difficult to perform, this technique is nevertheless essential in the analytical modeling and evaluation of optical systems. Traditional skew ray tracing, even when performed with computers, is computationally expensive. A previous study by the current authors expedited the skew ray tracing procedure by formulating the process in terms of homogeneous transformation matrices. The resulting ray tracing operation was then applied to the simply-manufactured flat and spherical optical components commonly used in industry [1,2]. However, in some situations, aspherical boundary surfaces such as ellipsoids provide significant advantages compared to spherical boundary surfaces. For example, it is known that parallel incident rays reflected by a spherical concave mirror undergo spherical aberration and do not converge at the focal point. However, a convex/concave ellipsoidal

---

* Corresponding author.
E-mail address: cy1006@seed.net.tw (P.D. Lin).

0307-904X/$ - see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2007.09.022

reflecting mirror can converge these rays to form a virtual/real-imaged point. Exploiting this effect, the Cassegrain and Gregorian telescope configurations both utilize convex secondary mirrors with ellipsoidal boundary surfaces.

Ray tracing for an aspherical surface is difficult since the intersection of a ray and an aspherical surface cannot be determined directly. Smith [3] performed aspherical-boundary skew ray tracing via a series of approximations which continued until the approximation error became negligible. However, this procedure was computationally expensive. The present study develops a more efficient aspherical-boundary skew ray tracing procedure by formulating Snell's laws as homogeneous transformation matrices, as shown in Section 2. A sensitivity analysis is presented in Section 3. A numerical comparison of the ray paths refracted through ellipsoidal and spherical boundary surfaces is also presented in these two sections.

The discussion presented in Sections 2 and 3 are limited to the case of monochromatic light. However, in practice, light sources are generally polychromatic. When polychromatic light is refracted, each monochromatic component has its own unique interaction with the refractive components of the optical system. Hence, each component follows a different ray path through the system and therefore arrives at a slightly different position. Consequently, in a phenomenon referred to as chromatic aberration, the resulting image is different for different colors. Welford [4] pointed out that the exact formulae for chromatic aberration are cumbersome. Nevertheless, Welford's numerical ray-tracing technique remains the universally adopted method for performing the detailed analysis of optical systems. However, even when performed with computers and commercial software, the ray-tracing process is slow. Therefore, Section 4 develops algebraic expressions for chromatic aberration which facilitate the rapid computerized evaluation of the quality of an optical system. Finally, the conclusions of the current study are presented in Section 5.

The 4×4 homogeneous transformation matrix is one of the most efficient and useful tools employed in the robotics [5,6], mechanisms [7,8], and computer graphics fields [9]. The notation of the position vector ${}^{j}\mathbf{P}_i$, unit direction vector ${}^{j}\ell_i$ and pose matrix ${}^{k}\mathbf{A}_j$ employed in this paper are expressed in homogeneous coordinate forms [5].

2. Skew ray tracing at ellipsoidal boundary surfaces

A common feature of typical optical components is that they possess boundary surfaces that can be described by revolution geometry. Therefore, the first step when determining a skew ray path is to establish the boundary surfaces of the optical components. An optical boundary surface of revolution ${}^{i}\mathbf{r}_i$ can be

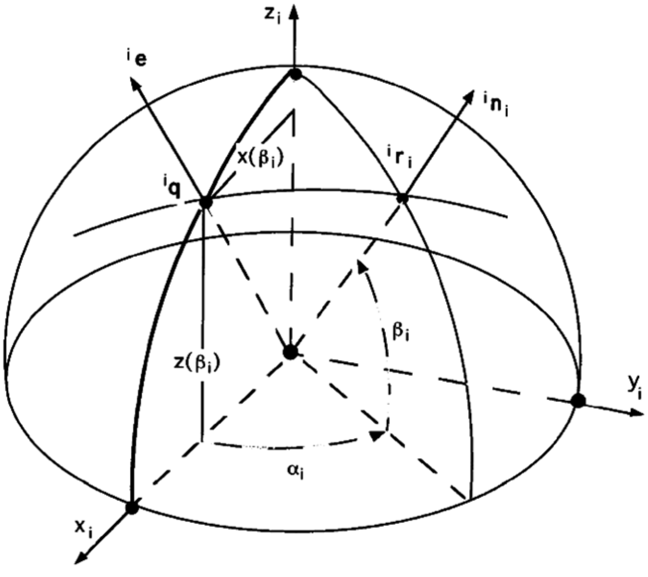

Fig. 1. The generating curve and its unit normal of an optical boundary surface.