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Sensitivity analysis of LP-MPC cascade control systems

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ABSTRACT

Model predictive control (MPC) has found wide application in the chemical process industry as well as other industrial sectors. Commercial MPC systems are typically implemented in conjunction with a steady-state linear or quadratic programming optimizer, whose key functions are to track the economic optimum and to provide feasible set-points to the model predictive controller. The two-level system is complementary to real-time optimization which typically utilizes more complex models and is executed less frequently. Despite the widespread adoption of LP-MPC systems, occurrences of poor performance have been reported, where large variations in the computed set-points were observed. In this paper, we analyze the sensitivity of the LP solution to variation in the LP model bias, through which feedback to the LP layer occurs. We consider both multi-input, single-output (MISO) and multi-input, multi-output (MIMO) systems. Principles are illustrated through graphical representation as well as case studies. The performance of the two-level LP-MPC closed-loop system is evaluated and explained using results of the LP sensitivity analysis.

1. Introduction

Model predictive control (MPC) is arguably the advanced control algorithm of choice in the chemical process industry, and has made inroads into other industrial sectors as well [10,13]. A dynamic model is utilized within the control algorithm to predict the effect of future plant inputs on the controlled outputs. Future inputs are computed in accordance with a performance objective, typically as the solution of an optimization problem. The inputs corresponding to the first control interval are implemented, and the calculation process repeated at the end of the sampling period, with the model predictions adjusted using the difference between the measured and predicted outputs. Details of the algorithm may be found, inter alia, in [4,9,13].

Industrial MPC systems are generally implemented in conjunction with a linear programming (LP) or quadratic programming (QP) steady-state optimizer [5,13,17,19,20]. The LP (or QP) typically uses a static model consistent with the dynamic MPC model, and is executed at the same frequency as the model predictive controller. The plant economic optimum may shift due to disturbances; thus the steady-state LP (QP) provides a bridge between a higher-level and less frequently executed real-time optimization (RTO) layer and the model predictive controller by making set-point adjustments in response to changing conditions between RTO executions. The LP formulation may involve minimization of the deviation between the set-points and target values determined by the real-time optimizer, or optimization of an economic criterion directly.

Fig. 1 illustrates the location of the steady-state LP (QP) within a plant automation hierarchy, as given in [19]. We note that the process would typically include local PID-type controllers and that several variants are possible, such as the presence of a plant unit optimization layer between the LP and plant-wide RTO layers [5,13].

Despite the apparent success of two-level LP-MPC systems, instances of poor performance have been reported [6,16]. Shah et al.[16], in the context of control performance monitoring, describe an industrial application in which the variation in the set-points exceeds that of the corresponding controlled variables. Kozub[6] also reports set-points being noisy relative to their controlled variables, and excessive variation in the set of inputs which are at their constraints. This motivates an investigation into the potential causes of such behavior.

Ying and Joseph [19] provide stability theorems for LP-MPC and QP-MPC cascade control systems with no plant/model mismatch. Consideration of model uncertainty is included in a case study based on the Shell Standard Control Problem [12]. Kassmann et al.[5] present a formulation for robust steady-state target calculation. For elliptic uncertainty on the model parameters, the target calculation problem takes the form of a second order cone program (SOCP) which the authors solve using software based on a primal-dual interior point algorithm. Steady-state targets are also computed in [11,14]. However, this is driven primarily not by economics, but rather to provide steady-state values for the plant inputs and states for inclusion in an MPC formulation in which

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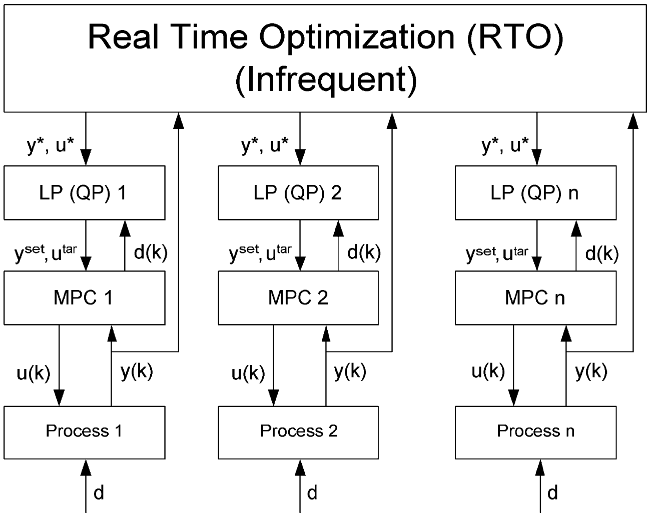

Fig. 1. LP (QP) within process automation hierarchy [19].

deviations of the states and inputs from corresponding steady-states are weighted by positive definite matrices.

Feedback in the two-level LP-MPC configuration occurs through a bias term in the steady-state LP model. In this paper, we analyze the sensitivity of the LP solution to variations in the LP model bias. We consider first the LP level separately for multi-input, single-output (MISO) and multi-input, multi-output (MIMO) systems. This analysis is coupled with graphical representations to provide insights into the sensitivity effects. Thereafter, the performance of a two-level LP-MPC cascade system is evaluated, with observed performance related to earlier sensitivity results.

2. Problem definition

The control structure we consider in this work is a two-layer LP-MPC system in which the LP provides output set-points and/or input target values to the model predictive controller. The MPC-controlled system feeds back information to the LP layer, from which model adjustments may be determined. We present in this section the MPC and LP formulations used, and also describe the LP model bias update method.

2.1. MPC

The model predictive control implementation we use here is quadratic dynamic matrix control (QDMC) [3], using a state-space formulation of the internal dynamic model [9]. The model predictions take the form,

$$\hat{\mathbf{x}}(k+i \mid k) = A\hat{\mathbf{x}}(k+i-1 \mid k) + B\hat{\mathbf{u}}(k+i-1 \mid k), \quad i = 1,\ldots,M \quad (1a)$$

$$\hat{\mathbf{x}}(k+i \mid k) = A\hat{\mathbf{x}}(k+i-1 \mid k) + B\hat{\mathbf{u}}(k+M-1 \mid k), \quad i = M+1,\ldots,P \quad (1b)$$

$$\hat{\mathbf{y}}(k+i \mid k) = C\hat{\mathbf{x}}(k+i \mid k) + \hat{\mathbf{d}}(k+i \mid k), \quad i = 1,\ldots P, \quad (1c)$$

where P is the prediction horizon, M is the control move horizon, $\hat{\mathbf{y}}(k+i \mid k) \in \Re^p$ represents the predicted values of the outputs at time step $k+i$ based on information available at time step k, and the predicted states, $\hat{\mathbf{x}}(k+i \mid k) \in \Re^n$, and inputs, $\hat{\mathbf{u}}(k+i \mid k) \in \Re^m$, are similarly defined. $A \in \Re^{n\times n}, B \in \Re^{n\times m}$ and $C \in \Re^{p\times n}$ are model coefficient matrices.

The disturbance estimate in the originally proposed Dynamic Matrix Control (DMC) and QDMC algorithms [1,3] is computed as the difference between the measured and predicted outputs, and assumed constant over the prediction horizon. In the present framework, this becomes:

$$\hat{\mathbf{d}}(k \mid k) = \mathbf{y}(k) - C\hat{\mathbf{x}}(k \mid k-1)$$
$$\hat{\mathbf{d}}(k+i \mid k) = \hat{\mathbf{d}}(k \mid k), \quad i = 1,\ldots,P$$

where

$$\hat{\mathbf{x}}(k \mid k) = \hat{\mathbf{x}}(k \mid k-1) = A\hat{\mathbf{x}}(k-1 \mid k-1) + B\mathbf{u}(k-1).$$

The above estimation scheme is based on an assumption of step-like disturbances, and may result in poor control performance in the presence of measurement noise and certain other disturbance types. A more general state estimation framework that addresses these limitations is described in [7–9,15]. In the application studies that follow, we consider only the original DMC disturbance estimation scheme.

2.2. LP

The set-points are determined through the solution of an LP of the form

$$\begin{aligned} \min_{\bar{\mathbf{u}},\bar{\mathbf{y}}} \quad & \sum_{i=1}^{p} \alpha_i \bar{y}_i + \sum_{i=1}^{m} \beta_i \bar{u}_i \\ s.t. \quad & \bar{\mathbf{y}} = G^{ss}\bar{\mathbf{u}} + \mathbf{d} \\ & \mathbf{y}^{\min} \leqslant \bar{\mathbf{y}} \leqslant \mathbf{y}^{\max} \\ & \mathbf{u}^{\min} \leqslant \bar{\mathbf{u}} \leqslant \mathbf{u}^{\max} \end{aligned} \quad (2)$$

where G^{ss} is the plant model gain matrix and $\mathbf{d}$ is a model bias that is updated in accordance with the actual plant outputs. We use the overbar to identify variables involved in the LP optimization. Components of the optimal $\bar{\mathbf{u}}$ and $\bar{\mathbf{y}}$ become the MPC set-points (including input target values). We typically assign a number of set-points equal to the number of manipulated inputs; specification of an excess number of set-points would in general result in offset.

In real-time optimization, the bias update, $\mathbf{d}$, is typically calculated as the difference between the measured outputs and those resulting from applying the plant inputs to the steady-state model [2]. In the present framework, this would correspond to

$$\mathbf{d} = \mathbf{y} - G^{ss}\mathbf{u},$$

where $\mathbf{y}$ and $\mathbf{u}$ correspond to the steady-state plant outputs and inputs respectively. For higher LP execution frequencies than transitions between steady-states, the MPC disturbance estimate may be used [19],

$$\mathbf{d} = \mathbf{y} - \hat{\mathbf{y}}(k \mid k-1).$$

3. Sensitivity analysis: MISO systems

For simplicity of analysis, a multi-input, single-output (MISO) system is considered first; such a system has only one bias term in its formulation. The LP then has the form:

$$\begin{aligned} \min_{\bar{\mathbf{u}},\bar{y}} \; & \alpha\bar{y} + \sum_{i=1}^{m} \beta_i \bar{u}_i \\ \text{s.t.} \quad & \bar{y} = \sum_{i=1}^{m} g_i^{ss}\bar{u}_i + d \\ & y^{\min} \leqslant \bar{y} \leqslant y^{\max} \\ & u_i^{\min} \leqslant \bar{u}_i \leqslant u_i^{\max}, \quad i = 1,\ldots,m \end{aligned} \quad (3)$$

where α and β_i are price coefficients and g_i^{ss} are steady-state gains of the process. According to the properties of an LP, a finite solution to problem (3) lies at the boundary of the feasible region, which is determined by the inequality constraints. For fixed cost function coefficients and optimization variable bounds, the particular location of the solution depends only on the value of the bias d. From this viewpoint, the LP problem can be considered as a mapping from $\Re^1 \to \Re^{m+1}$, where $m+1$ is the number of optimization variables.