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Estimating thermal contact resistance using sensitivity analysis and regularization

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ABSTRACT

Characterization of the thermal contact resistance is important in modeling of multi-component thermal systems which feature mechanically mated surfaces. Thermal resistance is phenomenologically quite complex and depends on many parameters including surface characteristics of the interfacial region and contact pressure. Although most studies seek a single value as a function of these parameters, in general, the contact resistance is non-uniform over the interface. In this paper, a technique is developed for extracting non-uniform contact resistance values from experiments in two-dimensional configurations. To begin, a two-dimensional model problem is formulated for a known contact resistance between two mated surfaces. An inverse problem is devised to estimate the variation of the contact resistance by using the BEM to determine sensitivity coefficients for specific temperature measurement points in the geometry. Temperature measured at these discrete locations can be processed to yield the contact resistance between the two mating surfaces using a simple matrix inversion technique. The inversion process is sensitive to noise and requires using a regularization technique to obtain physically possible results. The regularization technique is then extended to a genetic algorithm for performing the inverse analysis. Numerical simulations are carried out to demonstrate the approach. Random noise is used to simulate the effect of input uncertainties in measured temperatures at the sensors.

1. Introduction

Thermal systems generally feature composite regions that are mechanically mated. There exists an often significant temperature drop across the interface between such regions which may be composed of similar or different materials. The parameter characterizing this temperature drop is the thermal contact resistance, $R''_{t,c} = \Delta T/q''$, which is defined as the ratio of the temperature drop, ΔT, to the heat flux normal to the interface, q''. The thermal contact resistance is due to roughness effects between mating surfaces which cause certain regions of the mating surfaces to lose contact thereby creating gaps. In these gap regions, the principal modes of heat transfer are conduction across the fluid filling the gap and radiation across the gap surfaces. Moreover, the contact resistance is a function of contact pressure as this can significantly alter the topology of the contact region. Clearly, the thermal contact resistance is a phenomenologically complex function and can significantly alter prediction of thermal models of complex multi-component structures. Accurate estimates of thermal contact resistance are thus important in engineering calculations and find application in thermal analysis ranging from relatively simple layered and composite materials to more complex biomaterials. There have been many studies devoted to the theoretical predictions of thermal contact resistance for instance [1–4] and comprehensive reviews of previous work on thermal contact resistance can be found in [5–8]. Although general theories have been somewhat successful in predicting thermal contact resistances, most reliable results have been obtained experimentally. This is due to the fact that the nature of thermal contact resistance is quite complex and depends on many parameters including types of mating materials, surface characteristics of the interfacial region such as roughness and hardness, and contact pressure distribution. In experiments, temperatures are measured at a certain number of locations, usually close to the contact surface, and these measurements are used as inputs to a parameter estimation procedure to arrive at the sought-after thermal contact resistance. Most studies seek a single value for the contact resistance, while the resistance may in fact also vary spatially.

In this paper, an inverse problem [9–11] is formulated to estimate the variation of the thermal contact resistance along an interface in a two-dimensional configuration. Temperature measured at discrete locations using embedded sensors placed in proximity to the interface provide the information required to

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solve the inverse problem. The contact resistance is found by using a superposition method to determine sensitivity coefficients [12,13] for specific temperature measurement points in the geometry. This serves to guide in the location of the measuring points. Temperature measured at these discrete locations are then used in a regularized least-squares problem to yield the contact resistance between the two mating surfaces. A boundary element method (BEM) [14–19] is also used to solve for the temperature under current estimates of the contact resistance during the solution of the inverse problem. The inverse problem is solved using sensitivity analysis and also via a regularized BEM/genetic algorithm (GA) [20] approach previously developed by the authors [21]. The L-curve method of Hansen [22,23] is used to choose the optimal regularization parameter. A series of numerical examples are provided to demonstrate the approach.

2. Problem definition

In general, the thermal contact resistance may vary spatially along the contact surface between mating regions. Most studies neglect that variation and rather seek a single value of the contact resistance as a function of certain parameters, such as temperature, contact pressure, and interfacial fluid. In this paper, we consider an intermediate model, whereby the contact resistance is assumed to vary with position along a contact line in a two-dimensional configuration. We simulate a steady-state experiment seeking to characterize the unknown variation of the contact resistance.

The general configuration of interest is a two-dimensional region illustrated in Fig. 1. Two blocks of materials of different conductivities k_u and k_b are joined at an interface located at a level $y = l$. The top and bottom surfaces of the sample are kept at constant but different temperatures, and the sides are modeled as adiabatic. Temperature measurements are carried out at a series of locations close to the interface, here, illustrated by solid dots in Fig. 1 and the purpose of the inverse problem is to identify the functional variation of $R''_{t,c}$.

In solving the inverse problem, a means of solving a forward problem given current estimates of the sought-after quantity(ies), contact resistance variation in our case, must be available. For this

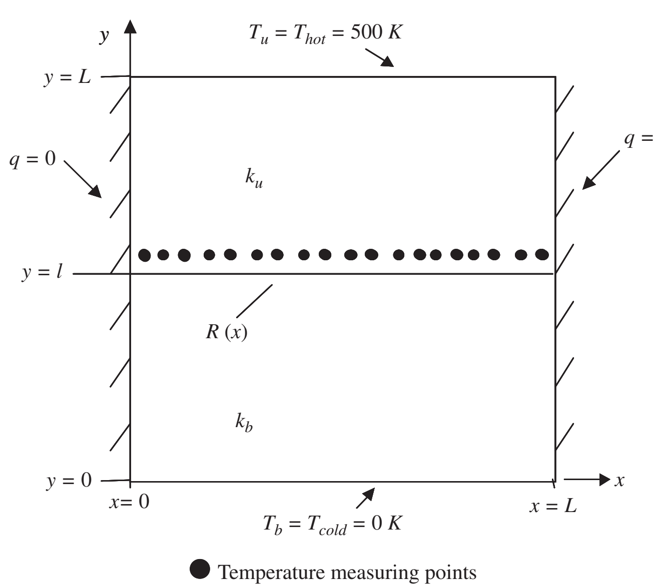

Fig. 1. Illustration of the model of a simulated steady-state experiment to retrieve thermal contact resistance.

purpose, we utilize an analytical solution along with a numerical solution based on the BEM. The forward problem for the problem illustrated above is formulated considering the upper block temperature $T_u(x, y)$ and the lower block temperature $T_b(x, y)$:

$$
\begin{aligned}
&\text{(a) } \nabla^2 T_u = 0 \\
&\text{(b) } \nabla^2 T_b = 0 \\
&\text{(c) } \frac{\partial T_u(0,y)}{\partial x} = \frac{\partial T_u(L,y)}{\partial x} = \frac{\partial T_b(0,y)}{\partial x} = \frac{\partial T_b(L,y)}{\partial x} = 0 \\
&\text{(d) } k_u \frac{\partial T_u(x,y)}{\partial y} = k_b \frac{\partial T_b(x,y)}{\partial y}\bigg|_{y=l} \\
&\text{(e) } k_b \frac{\partial T_b(x,l)}{\partial y} = \frac{T_u(x,l) - T_b(x,l)}{R''_{t,c}(x)}
\end{aligned}
\tag{1}
$$

where $R''_{t,c}(x)$ is the spatially varying contact resistance per unit area. An analytical solution can be derived for the temperature distribution on the upper block and on the lower block by satisfying the governing equations and boundary conditions specified in Eq. (1.a) through Eq. (1.d). The analytical solution can take into account a variety of spatial functions $R_{t,c}(x)$ and can be used to verify the BEM numerical solution we used later in our analysis [21].

3. Boundary element model for conduction with contact resistance

The BEM is a numerical implementation of boundary integral methods for solution of field problems. The BEM is now a well-established numerical method which can be efficiently used to solve heat conduction problems in linear and nonlinear media as well as non-homogeneous media using boundary-only discretization. In addition to boundary-only discretization, a distinct feature of BEM is that unknowns which appear in the BEM formulation are the surface temperature and heat flux.

Assuming that the conductivity is constant, the Laplace equation governs the temperature field in each region in Fig. 1. Should the conductivity significantly vary as a function of temperature, the Kirchhoff transform can be used to linearize the heat conduction equation to the Laplace equation in the Kirchhoff transform [14,15]. In the direct BEM used in this paper, the governing equation is first converted to a boundary integral equation (BIE) by: (1) multiplying the governing equation by a test function $G(x, \xi)$, (2) integrating over the spatial domain and using Green's second identity, and (3) invoking properties of the Green free-space solution identified as the test function $G(x, \xi)$, resulting in

$$
C(\xi)T(\xi) = \oint_\Gamma [H(x,\xi)T(x) - q(x)G(x,\xi)]\,\mathrm{d}\Gamma \tag{2}
$$

This BIE is valid for boundary or interior points. Here, Γ is the domain boundary of a domain Ω, $H(x, \xi) = -k\,\partial G(x, \xi)/\partial n$, $q(x, t) = -k\,\partial T(x)/\partial n$, and $\partial/\partial n$ denotes the normal derivative with respect to the outward-drawn normal. The free term, $C(\xi)$ is 1, for $\xi \in \Omega$ and is equal to the internal angle subtended at a point on the boundary, $\xi \in \Gamma$, divided by 2π radians in two dimension and 4π steradian in three dimension. The Green free-space solution for the Laplace equation, solves the adjoined diffusion equation perturbed in free space by a Dirac delta function located at the source point $x = \xi$ and is, $G(x, \xi) = -\ln r/2\pi k$ in two dimension with $r = |x - \xi|$. The boundary Γ is discretized using N-boundary elements, and the flux and temperature are discretized over the boundary to lead to the following form:

$$
C_i T_i + \sum_{j=1}^{N} H_{ij} T_j = \sum_{j=1}^{N} G_{ij} q_j \tag{3}
$$