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Advanced types of the sensitivity analysis in frequency and time domains

Josef Dobeš*

Department of Radio Engineering, Czech Technical University of Prague, Technická 2, 166 27 Praha 6, Czech Republic

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Abstract

The majority of standard tools for computer-aided design can perform only limited types of the sensitivity analysis: Spice determines operating-point sensitivities, and SpectreRF contains a parametric sensitivity analysis that can be used for determining the phase noise, for example. In the paper, some new types of the sensitivity analysis in frequency and time domains are described. These types of the analysis are not implemented in the standard circuit simulators.

In the *frequency domain*, a procedure for determining the sensitivities of the noise figure is suggested. First, an improvement of the method for computing the noise figure is presented, which incorporates necessary circuit matching and eliminates the subtraction of output noise from the load at each frequency. Second, a simple formula is derived for computing the sensitivities of the noise figure. The sensitivity analysis in the frequency domain is generally demonstrated by means of a distributed microwave amplifier. The application of the sensitivity analysis of the noise figure for improving the noise properties of a monolithic microwave amplifier is described.

In the *time domain*, a new recurrent formula is derived for the sensitivity analysis that efficiently uses high-order expressions of the algorithm for implicit numerical integration. Since the chosen integration algorithm is more flexible than the more frequently used Gear's one, the suggested formula leads to more efficient procedure. The sensitivity analysis in the time domain is important for analyzing symmetrical microwave circuits, because their operating-point sensitivities are zero in principle. For this reason, the significance of the proposed method is demonstrated by an analysis of a symmetrical radio-frequency CMOS multiplier. As an unusual example of exploiting the method, a temperature sensitivity analysis of a power operational amplifier is described.

Keywords: Computer-aided design; Sensitivity analysis; Noise figure; Implicit numerical integration; Temperature analysis; Four-quadrant multiplier

1. Introduction to the frequency domain

The Affirma RF circuit simulator [1] (it is also known as SpectreRF) contains a special type of the sensitivity analysis in the frequency domain, which models the frequency translation, and it can determine the sensitivity of the output to either up-converted or down-converted noise from power supplies or a local oscillator. In this section, the sensitivity analysis in the frequency domain is generally described, and a procedure for computing the sensitivities of the noise figure is defined subsequently.

The sensitivity analysis in the frequency domain should use computational by-products of a conventional AC analysis, especially $\boldsymbol{L}$ and $\boldsymbol{U}$ matrices resulting from complex LU factorizations. For this reason, the sensitivity analysis (including the sensitivity analysis of the noise figure) is performed as a part of the AC analysis. The formulae of the sensitivity analysis have been implemented into an original software tool Circuit Interactive Analyzer (C.I.A.) [2].

* Tel.: +420 2 2435 2207; fax: +420 2 3333 9801.
E-mail address: dobes@feld.cvut.cz.

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2. Sensitivity analysis in the frequency domain

A parametric system of the circuit linear equations (or the equations that are linearized at the circuit operating point) can be written in the form

$$\boldsymbol{A}(p)\boldsymbol{x}(p)=\boldsymbol{b}(p), \tag{1}$$

where p is one of the circuit parameters to which the sensitivities are requested. The vector of the derivatives with respect to this parameter $\partial\boldsymbol{x}(p)/\partial p$ marked for simplicity by $\boldsymbol{x}'(p)$ can be obtained by differentiating (1)

$$\boldsymbol{A}'(p)\boldsymbol{x}(p)+\boldsymbol{A}(p)\boldsymbol{x}'(p)=\boldsymbol{b}'(p), \tag{2}$$

which gives the basic system of the complex linear equations

$$\boldsymbol{A}(p)\boldsymbol{x}'(p)=\boldsymbol{b}'(p)-\boldsymbol{A}'(p)\boldsymbol{x}(p). \tag{3}$$

2.1. Standard sensitivity analysis

The circuit contains an independent input source and no other internal sources in this case. Therefore, the first part of the right side of (3) is equal to zero and the system is simpler

$$\boldsymbol{A}(p)\boldsymbol{x}'(p)=-\boldsymbol{A}'(p)\boldsymbol{x}(p). \tag{4}$$

If simulator procedures are unable to determine the parametric derivatives $\partial A_{\ldots}/\partial p$ symbolically, they must be computed numerically. For example, the natural approximation of the derivatives

$$\boldsymbol{A}'(p)\approx\frac{\boldsymbol{A}(p+\Delta p)-\boldsymbol{A}(p)}{\Delta p} \tag{5}$$

can be used, and inserting this formula into (4) considering (1) gives the system

$$\boldsymbol{A}(p)\boldsymbol{x}'(p)=\frac{\boldsymbol{b}(p)-\boldsymbol{A}(p+\Delta p)\boldsymbol{x}(p)}{\Delta p}. \tag{6}$$

The real and imaginary parts obtained by the solution of the exact system (4) or its approximation (6) serve for creating other useful forms of the output of the sensitivity analysis

$$x_i^{\text{magnitude}}=\sqrt{{}^{\Re}x_i^2+{}^{\Im}x_i^2}\;\Rightarrow\; x_i^{\text{magnitude}\prime}=\frac{{}^{\Re}x_i\,{}^{\Re}x_i'+{}^{\Im}x_i\,{}^{\Im}x_i'}{\sqrt{{}^{\Re}x_i^2+{}^{\Im}x_i^2}}, \tag{7a}$$

$$x_i^{\text{dB}}=10\log\left({}^{\Re}x_i^2+{}^{\Im}x_i^2\right)\;\Rightarrow\; x_i^{\text{dB}\prime}=\frac{20}{\ln(10)}\,\frac{{}^{\Re}x_i{}^{\Re}x_i'+{}^{\Im}x_i{}^{\Im}x_i'}{{}^{\Re}x_i^2+{}^{\Im}x_i^2}, \tag{7b}$$

$$x_i^{\text{arg}}=\frac{180}{\pi}\arctan\left(\frac{{}^{\Im}x_i}{{}^{\Re}x_i}\right)\;\Rightarrow\; x_i^{\text{arg}\prime}=\frac{180}{\pi}\,\frac{{}^{\Re}x_i{}^{\Im}x_i'-{}^{\Im}x_i{}^{\Re}x_i'}{{}^{\Re}x_i^2+{}^{\Im}x_i^2} \tag{7c}$$

for $i=1,\ldots,n_x$, where n_x is the dimension of the vector $\boldsymbol{x}$, and $\Re$ and $\Im$ mark the real and imaginary parts, respectively.

2.2. Noise sensitivity analysis

The circuit contains n_n internal noise sources and no independent input source in this case. A jth output of the noise analysis is determined by solving the system

$$\boldsymbol{A}(p)\,{}_j\boldsymbol{x}(p)={}_j\boldsymbol{b}(p),\quad j=1,\ldots,n_n, \tag{8}$$

which is of the same type as (1). Therefore, the complex LU factorization of $\boldsymbol{A}$ must be executed only once for each frequency, which is conventional and very important, of course. Similarly, the sensitivity of the jth output is determined by solving the system

$$\boldsymbol{A}(p)\,{}_j\boldsymbol{x}'(p)={}_j\boldsymbol{b}'(p)-\boldsymbol{A}'(p)\,{}_j\boldsymbol{x}(p). \tag{9}$$

If the procedure is unable to compute the parametric derivatives $\partial A_{\ldots}/\partial p$ symbolically, they must be determined numerically using the approximation (5) and the similar one

$${}_j\boldsymbol{b}'(p)\approx\frac{{}_j\boldsymbol{b}(p+\Delta p)-{}_j\boldsymbol{b}(p)}{\Delta p},\quad j=1,\ldots,n_n, \tag{10}$$

which gives the approximation of (9)

$$\boldsymbol{A}(p)\,{}_j\boldsymbol{x}'(p)=\frac{{}_j\boldsymbol{b}(p+\Delta p)-\boldsymbol{A}(p+\Delta p)\,{}_j\boldsymbol{x}(p)}{\Delta p},\quad j=1,\ldots,n_n. \tag{11}$$

An ith component x_i^{noise} of the output of the noise analysis $\boldsymbol{x}^{\text{noise}}$ and its sensitivity $x_i^{\text{noise}\prime}$ are determined by the solutions of the exact systems (8) and (9), or by the solutions of the exact system (8) and the approximation (11)

$$x_i^{\text{noise}}=\sqrt{\sum_{j=1}^{n_n}\left({}_j^{\Re}x_i^2+{}_j^{\Im}x_i^2\right)}\;\Rightarrow\; x_i^{\text{noise}\prime}=\frac{\sum_{j=1}^{n_n}\left({}_j^{\Re}x_i\,{}_j^{\Re}x_i'+{}_j^{\Im}x_i\,{}_j^{\Im}x_i'\right)}{x_i^{\text{noise}}}. \tag{12}$$

It is necessary to point out that the standard noise models of transistors in Spice do not take advantage of the noise correlation, which means that the influence of S_{12} is not considered in the calculations. Therefore, the use of (12) is limited only to the frequencies below f_T of the transistors.

2.3. Sensitivity analysis of the noise figure

General formulae for computing the noise factor F_n and the noise figure F_n^{dB} have been derived in [3] (see also Fig. 4)

$$F_n=\frac{V_n^2-V_{n,R_{\text{load}}}^2}{A_V^2 4kT_0R_{\text{source}}},\quad F_n^{\text{dB}}=10\log(F_n), \tag{13}$$