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Subset simulation for structural reliability sensitivity analysis

Shufang Song, Zhenzhou Lu *, Hongwei Qiao

School of Aeronautics, Northwestern Polytechnical University, Xi'an, Shaanxi, 710072, PR China

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ABSTRACT

Based on two procedures for efficiently generating conditional samples, i.e. Markov chain Monte Carlo (MCMC) simulation and importance sampling (IS), two reliability sensitivity (RS) algorithms are presented. On the basis of reliability analysis of Subset simulation (Subsim), the RS of the failure probability with respect to the distribution parameter of the basic variable is transformed as a set of RS of conditional failure probabilities with respect to the distribution parameter of the basic variable. By use of the conditional samples generated by MCMC simulation and IS, procedures are established to estimate the RS of the conditional failure probabilities. The formulae of the RS estimator, its variance and its coefficient of variation are derived in detail. The results of the illustrations show high efficiency and high precision of the presented algorithms, and it is suitable for highly nonlinear limit state equation and structural system with single and multiple failure modes.

1. Introduction

Sensitivity analysis has been widely applied in a broad range of sciences, for example, chemistry, environmental sciences, aerospace engineering, risk analysis, etc. In engineering design, sensitivity analysis explores the model response, evaluates the accuracy of model, tests the validity of the assumption made and so on. Sensitivity is used to find the rate of change in a model output due to changes in the model inputs in deterministic design, which is usually performed by partial derivative analytically or numerically.

Random uncertainty exists in the model parameter of system widely due to uncontrollable factors. When random uncertainty is considered, sensitivity analysis has different meanings. We assume that the random uncertainty in a design performance is described probabilistically by its mean (μ), variance (σ), probability density function (PDF) or cumulative distribution function (CDF), etc. Correspondingly, the sensitivity analysis under uncertainty needs to be performed on the probability characteristics of a model response with respect to the probability characteristics of the model inputs. In general, the reliability sensitivity analysis is to investigate the rate of change in the probability characteristics of the response, especially failure probability P_f, due to the probability characteristics of a basic random variable x_i, such as $\partial P_f/\partial \mu_{x_i}$ and $\partial P_f/\partial \sigma_{x_i}$ [1–7]. These partial derivatives can objectively describe the effect of distribution parameters on the failure probability. And this paper studies the partial derivative of the failure probability with respect to the distribution parameter of the basic random variable.

Wu [1,2] presented a reliability sensitivity method based on the CDF of the structural response variable. The normalized reliability sensitivity coefficient is expressed as an expectation of the partial derivative of the PDF, evaluated over the failure region, wherein the sampling based method can be used to compute the reliability sensitivity.

The reliability sensitivity based on the most probable point (MPP) is very simple, but it depends on the linearization of the limit state function [6,7]. The linearization of the nonlinear performance function will lead to a weak precision in the reliability and the reliability sensitivity evaluations.

This paper focuses on the numerical method based on subset simulation (Subsim) to analyze the reliability sensitivity. Subsim [8–14] is an efficient simulation to perform the reliability analysis in a progressive manner. Introducing a set of intermediate failure events, Subsim separates the original probability space into a sequence of subsets, and then the small failure probability can be expressed as a product of larger conditional failure probabilities. Markov chain Monte Carlo (MCMC) simulation [8] is used to generate conditional samples that correspond to specified levels of failure probabilities. However, the conditional samples generated by MCMC are dependent in general. These samples are used for statistical averaging as if they are independent and identically distributed (i.i.d.) with some reduction in efficiency. The concept of importance sampling (IS) procedure is employed for generating i.i.d. conditional samples to efficiently calculate the conditional failure probability corresponding to the specified levels of failure probabilities [14]. On the basis of reliability analysis of Subsim/

* Corresponding author. Tel./fax: +86 29 88460480.
E-mail addresses: songshufangssf@mail.nwpu.edu.cn (S. Song), zhenzhoulu@nwpu.edu.cn (Z. Lu).

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MCMC and Subsim/IS, the RS of the failure probability with respect to the distribution parameter of the basic variable is transformed as a set of RS of conditional failure probabilities with respect to the distribution parameter of the basic variable, and the latter can be evaluated by the conditional samples generated by MCMC simulation and IS. The concept and the implementation of the presented Subsim-based reliability sensitivity are explained in the following sections.

The outline of the paper is as follows. The reliability analysis of Subsim/MCMC and Subsim/IS are described in Section 2. Section 3 explains the concept and the implementation of Subsim-based reliability sensitivity method. In Section 3.1, the basic concept of Subsim/MCMC-based reliability sensitivity method is described in brief. The reliability sensitivity estimator and its variance analysis based on Subsim/IS are discussed in Section 3.2. After the numerical examples verify the feasibility and rationality of the presented method in Section 4, Section 5 concludes with a summary of the main advantages of the presented method.

2. The reliability analysis of Subsim

2.1. The basic idea of Subsim [8–13]

In failure event $F=\{\boldsymbol{x}:g(\boldsymbol{x})<0\}$, $g(\boldsymbol{x})$ is a limit state function of the random variables $\boldsymbol{x}=(x_1,x_2,\ldots,x_n)$. The PDF of $\boldsymbol{x}$ is denoted by $f_X(\boldsymbol{x})$. Assume $b_1>b_2>\cdots>b_m=0$ as a decreasing sequence of the threshold values of failure events $F_k=\{\boldsymbol{x}:g(\boldsymbol{x})<b_k\}$ $(k=1,2,\ldots,m)$ shown in Fig. 1. Then the failure events satisfy the following relations:

$$F_1\supset F_2\supset\cdots\supset F_m=F$$

and

$$F_k=\bigcap_{i=1}^{k}F_i$$

According to the multiplication theorem and the definition of conditional probability in the probability theory, the following equation holds:

$$P_{\mathrm{f}}=P(F)=P(F_m|F_{m-1})P(F_{m-1})=\cdots=P(F_1)\prod_{i=2}^{m}P(F_i|F_{i-1}) \quad (1)$$

Eq. (1) expresses the failure probability as a product of a sequence of conditional probabilities $P(F_i|F_{i-1})$ $(i=2,3,\ldots,m)$ and $P(F_1)$. The idea of Subsim is to obtain the failure probability P_{f} by estimating these conditional probability quantities.

Define $P_1=P(F_1)$, $P_i=P(F_i|F_{i-1})$ $(i=2,\ldots,m)$. Failure probability in Eq. (1) can be expressed by

$$P_{\mathrm{f}}=\prod_{i=1}^{m}P_i \quad (2)$$

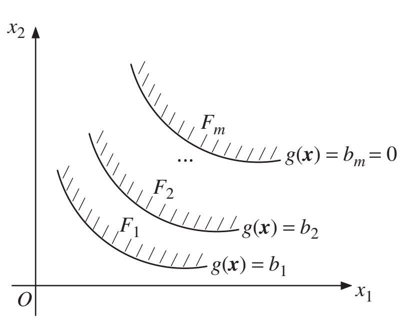

Fig. 1. A set of failure events in Subsim.

Generally, P_{f} is small in engineering and it cannot be estimated efficiently by numerical simulation. By choosing the intermediate failure events F_i $(i=1,2,\ldots,m-1)$ appropriately, conditional probabilities involved in Eq. (2) can be made sufficiently large so that they can be evaluated efficiently by simulation procedures. The problem of simulating rare events in the original probability space is thus replaced by a sequence of simulations of more frequent events in the conditional probability spaces.

In Eq. (2), P_1 can be estimated by direct Monte Carlo simulation (MCS):

$$\hat{P}_1=\frac{1}{N_1}\sum_{k=1}^{N_1}I_{F_1}\left[\boldsymbol{x}_k^{(1)}\right] \quad (3)$$

where $\boldsymbol{x}_k^{(1)}(k=1,2,\ldots,N_1)$ are i.i.d. samples drawn from the PDF $f_X(\boldsymbol{x})$. $I_{F_1}[\boldsymbol{x}_k^{(1)}]$ is an indicator function, when $\boldsymbol{x}_k^{(1)}\in F_1$, $I_{F_1}[\boldsymbol{x}_k^{(1)}]=1$, otherwise $I_{F_1}[\boldsymbol{x}_k^{(1)}]=0$.

Similarly, the conditional failure probabilities P_i $(i=2,3,\ldots,m)$ in Eq. (2) can be estimated by drawing samples from the conditional PDF $q(\boldsymbol{x}|F_{i-1})=I_{F_{i-1}}[\boldsymbol{x}]f_X(\boldsymbol{x})/P(F_{i-1})$:

$$\hat{P}_i=\frac{1}{N_i}\sum_{k=1}^{N_i}I_{F_i}[\boldsymbol{x}_k^{(i)}] \quad (i=2,3,\ldots,m) \quad (4)$$

where $\boldsymbol{x}_k^{(i)}$ $(k=1,2,\ldots,N_i;\ i=2,3,\ldots,m)$ are i.i.d. conditional samples drawn from the conditional PDF $q(\boldsymbol{x}|F_{i-1})$. $I_{F_i}[\boldsymbol{x}_k^{(i)}]$ is an indicator function that is equal to 1 when $\boldsymbol{x}_k^{(i)}\in F_i$ and is equal to zero otherwise.

Although, the direct MCS can be used to obtain conditional samples of the conditional PDF $q(\boldsymbol{x}|F_{i-1})$, it is not efficient since on average it should take $1/P(F_{i-1})$ samples before one conditional sample occurs. In general, the task of efficiently simulating conditional samples is not trivial by direct MCS. Efficient methods for estimating conditional failure probability are introduced in the next two subsections.

The choice of the intermediate failure events F_i $(i=1,2,\ldots,m-1)$ plays a key role in the Subsim procedure. The more the simulation levels m introduced (i.e. if the threshold values b_i of failure events decrease slowly) the larger the conditional failure probabilities, and fewer the samples that estimations require. Then, the total number of samples $N=\sum_{i=1}^{m}N_i$ is large in the whole procedure. Conversely, if the threshold values b_i decrease too rapidly that the conditional failure events become rare, to obtain an accurate estimate of the conditional failure probabilities in each simulation level more samples are required, which also increases the total number of samples. It can be seen that the choice of the intermediate failure events is a compromise between N_i, the number of samples required in each simulation level, and m, the number of simulation levels. An adaptive stratification procedure is employed to choose the intermediate failure events in the Subsim [12,13].

2.2. Subsim/MCMC

MCMC algorithm fits nicely into computing conditional failure probabilities by using Markov chain samples with limiting stationary distribution $q(\boldsymbol{x}|F_{i-1})$ $(i=2,3,\ldots,m)$. The Metropolis-Hastings Criterion is employed to draw the Markov chain samples. And the Subsim/MCMC proceeds as in the following:

(1) Generate N_1 i.i.d. samples $\boldsymbol{x}_k^{(1)}$ $(k=1,2,\ldots,N_1)$ of the PDF $f_X(\boldsymbol{x})$ by direct MCS for $i=1$.
(2) Compute the corresponding response values $g(\boldsymbol{x}_k^{(1)})$ $(k=1,2,\ldots,N_1)$. The first intermediate threshold value b_1 is adaptively chosen as the (p_0N_1)th (p_0 is a pre-established conditional