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Classical and imprecise probability methods for sensitivity analysis in engineering: A case study

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ABSTRACT

This article addresses questions of sensitivity of output values in engineering models with respect to variations in the input parameters. Such an analysis is an important ingredient in the assessment of the safety and reliability of structures. A major challenge in engineering applications lies in the fact that high computational costs have to be faced. Methods have to be developed that admit assertions about the sensitivity of the output with as few computations as possible. This article serves to explore various techniques from precise and imprecise probability theory that may contribute to achieving this goal. It is a case study using an aerospace engineering example and compares sensitivity analysis methods based on random sets, fuzzy sets, interval spreads simulated with the aid of the Cauchy distribution, and sensitivity indices calculated by direct Monte Carlo simulation. Computational cost, accuracy, interpretability, ability to incorporate correlated input and applicability to large scale problems will be discussed.

1. Introduction

The goal of this article is to demonstrate how various methods from probability theory and imprecise probability theory can be employed in sensitivity analysis of engineering structures. We are motivated by a research project in aerospace engineering[2] which involved the determination of the buckling load of the frontskirt of the ARIANE 5 launcher under various loading and flight scenarios. The frontskirt is a reinforced light weight shell structure. The computation of the decisive parameter indicating failure, the load proportionality factor (LPF), is based on a finite element model.[3] Part of the project was to determine the most influential input parameters (loads, material constants, geometry) on the load proportionality factor in a sensitivity

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[2] ICONA-project, Intales GmbH Engineering Solutions and University of Innsbruck, supported by TransIT Innsbruck and by EADS Astrium ST.

[3] The load proportionality factor is defined as the limiting value in an incremental procedure in which the dynamic loads during a flight scenario are increased step by step until breakdown of the structure is reached.

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analysis. The goal was to evaluate the design and to assess the safety of the structure. The calculation of the output variable LPF – under a given single set of input parameters – takes about 32 h on a high performance computer. In addition to the extremely high computational cost, the LPF may depend in a non-differentiable manner on some of the input parameters, especially variations in the geometry. A classical sensitivity analysis of the complete structure is currently out of reach.

Engineering information on the variability of the input parameters usually consists of a central value and a coefficient or range of variation. The basic strategy for arriving at a sensitivity assessment will be to successively pinch the input parameters (that is, freeze them at their central value) and study the effect on the variability of the output. Among the numerous alternative views on sensitivity analysis (cf. e.g. [14]), we shall also pursue simulation methods that yield interval bounds on the output as well as variance-based methods.

We wish to do our analysis without artificial parametric assumptions and with as few calls of the finite element program as possible. We will explore the usability of the following four methods, modelling the input variability by means of

- random sets and Tchebycheff's inequality;
- fuzzy sets and Hartley-like measures;
- intervals and sampling from a Cauchy distribution;
- standard Monte Carlo simulation and resampling.

Sensitivity analysis with the aid of Monte Carlo methods will include the computation of partial rank correlation coefficients and Sobol indices. The first three methods belong to imprecise probability in its proper sense; the last method is of a standard probabilistic type and included for comparison. Imprecise probability versions of the latter variance-based methods have been proposed by [10], but were not pursued in this study due to the expected additional computational costs.

A detailed description of the respective methods will follow in five sections, with a final section devoted to a comparison of the methods. The question of modelling correlations between the input variables will be addressed in the appropriate sections.

The ARIANE 5 frontskirt is the part of the launcher that connects the tanks section with the payload section and also has to support the booster loads. It consists of a light weight shell structure reinforced by struts. The full finite element model is composed of shell elements and solid elements, altogether with two million degrees of freedom. The load proportionality factor is computed by means of a path following procedure that follows bifurcations as long as possible until failure of the structure is reached, indicated by numerical breakdown of the program at a point at which the determinant of the local stiffness matrix does not change sign.

In order to make a test of the sensitivity analysis methods feasible, we made two simplifications. First, a simplified finite element model keeping the global structure (Fig. 1) with about ninety thousand degrees of freedom was used, and second, no distinction of bifurcation or material failure was made, so that the terminal value of the LPF was taken as that value at which the finite element program failed to converge. No further investigation of the reason for non-convergence was undertaken. The computational cost for the simplified model was one hour per call of the program.

In the sensitivity analysis, up to 17 input parameters were taken into account. A terse description of the meaning of the parameters as well as their nominal values can be read off from Table 1. This was all information we had available; in particular, no information on possible correlations of the input variables was given to us – actually on purpose; the project was in part a blind-folded test to check whether the sensitivity study would reveal dependencies which were expected from an engineering viewpoint (and it successfully did). The coefficients of variation of the input variables were estimated – after discussions with various engineering experts and consulting the literature [32] – at 15%.

At this point, it appears important to note that we did *not* intend to develop an imprecise probability model of the launcher. Rather, imprecise probability methods are introduced here for computing and visualizing sensitivities. So, while an imprecise probability model might well end up having different coefficients of variation for the various input parameters, it would be counter-productive to enter into a sensitivity study with different coefficients of variation, as this definitely would introduce distorting information (see also [28]).

To aid the reader in assessing the sensitivity results, we mention what we computed as the total variability of the output variable LPF: A direct Monte Carlo simulation of size $n = 100$ with input variables uniformly distributed on an interval around μ_i of spread $\pm 0.15\mu_i$ produced an output range of [3.45, 3.65] for the LPF.

For background material on sensitivity analysis we refer to the Special Issue [13], in particular the survey article [14] as well as [6,7,12]; for random sets, to [24,26]; for random and fuzzy sets, to [9,19]; for probability boxes, to [5]; for a review on

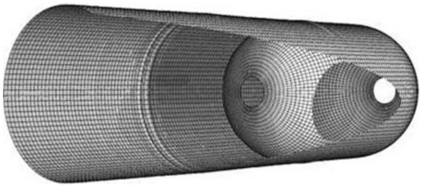

Fig. 1. Simplified finite element model of frontskirt.