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Modelling of Type I fracture network: Objective function formulation by fuzzy sensitivity analysis

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ABSTRACT

This paper advances the fundamental understanding in mathematical and computational modelling of discrete fracture networks (Type I). It presents a systematic procedure to solve the most important problem in modelling by global optimization — objective function formulation, which negates guesswork in objective function formulation by automatic selection of highly ranked components and their corresponding weighting factors. The procedure starts from real data to identify potential components of the objective function. The components are then ranked by fuzzy sensitivity analysis, based on their effects on the final objective function value and simulation convergence. The final fracture network inversion is subsequently realized and validated. Results of the study provide an explanation why previous methods such as stochastic simulations are not sufficiently reliable, compared to global optimization methods.

1. Introduction

Type I fracture networks refer to those where the rock matrix is non-porous and non-permeable and fractures dominate both storage and flowing capacities. Among underground natural resources, the type I fracture behaviours are most noticeable in fractured granite basement petroleum reservoirs and fractured sandstone geothermal sources. This type I performance is typically characterized by high production derivability at initial time and sudden/ rapid declines afterwards. Modelling discrete fractures in a type I fracture network is the essential first step in understanding the fluid storage and flowing mechanisms.

To date, there have been a wide range of approaches that model discrete fracture networks, among which global optimization methods (mostly simulated annealing) can be ranked as the most computationally and technologically advanced. They can provide fracture network inversions with certain success [1–5]. Recently, Tran et al. [6] further advance global optimization methodology by combining it with comprehensive fracture characterization, neuro-stochastic simulation, object-based and conditional modelling. The authors identify that the two determinant aspects of a successful global optimization model are: (1) an appropriate objective function, such that when minimized, the output is indeed a representation of the target; and (2) an efficient modification scheme. In the previous works, objective functions are chosen arbitrarily. Many components and measurements are not representative to a normal fracture system. Significant improvement has been made by combining the formulation of an objective function with comprehensive fracture characterization and by introducing non-parametric components [6]. However, it is realized that determining appropriate components and weighting factors for objective functions remain the major issue that limits applications of global optimization in practical geoscience modelling. This paper applies a fuzzy-logic-based sensitivity analysis to help ranking

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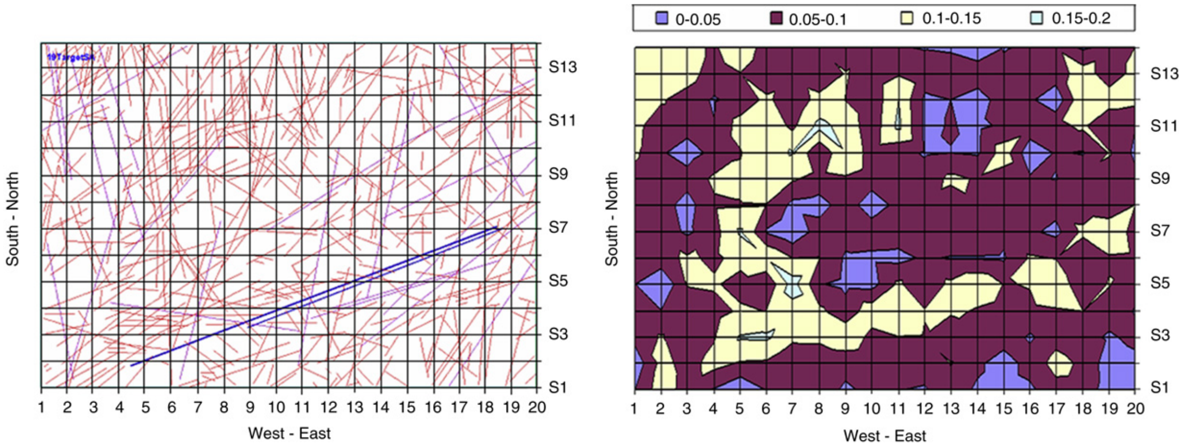

Fig. 1. Target fracture network map (and the corresponding fracture density map m/m^2. Lighter colours indicate denser fracture distribution).

components of an objective function based on their behaviour towards solution convergence. The procedure is illustrated using a fracture outcrop of the New York area as an example (Fig. 1).

2. Review of global optimization and fracture modelling

2.1. Global optimization of simulated annealing

Kirkpatrick et al. [7] introduced simulated annealing to complex problems in combinatorial optimization. Since then, it has been used in a variety of problems that involve finding optimum values for a function of a very large number of independent variables. The basic concept of simulated annealing originates from the physical process of metallurgical annealing. An annealing process occurs when a metal in a heat bath is initially at high temperature and is slowly cooled. At first, all particles are distributed randomly in a quasi-liquid state. As temperature drops, particles arrange themselves in a low energy ground state (i.e. at or very close to the global minimum of energy), forming a crystal. Model parameters of an optimization problem play the role of particles in an idealized physical system. Objective function is an analogue of the energy function which is to be minimized. It represents the mathematical difference between the newly generated state and target (*) (characterised fracture data from the actual field). It is composed of a weighted average of positive functions, as:

$$OF = \sum_i w_i \cdot \left[\frac{z_i - z_i^*}{z_i^*}\right]^2 \tag{1}$$

where z_i denotes arbitrary components of the objective function and w_i is the corresponding weighting factors ($\Sigma w_i = 1.0$). Starting from an initial state, the system is perturbed at random to a new state in its neighbourhood, from which the change (ΔOF) in value of the objective function can be quickly calculated. If this change represents a reduction in objective function, the transformation to new state is accepted. If it represents an increase in objective function, the transformation is accepted with a specified probability, $P(\text{accept})$:

$$P(accept) = \begin{cases} \exp\left(\dfrac{-\Delta OF}{T}\right) \leftarrow (\Delta OF > 0) \\ 1 \leftarrow \text{otherwise} \end{cases} \tag{2}$$

where T is a control parameter, which corresponds to absolute temperature in the physical process of annealing [8]. At a value of T, if the generation-acceptance progression is repeated a large number of times, it can be shown that a steady state is attained. By reducing T slowly (cooling), a new equilibrium (steady state) is achieved. Eventually, the system converges to a global optimum equilibrium that corresponds to the minimum objective function.

2.2. Simulated annealing and fracture modelling

Global optimization has effectively solved complex problems in combinatorial optimization, acquiring optimum values for functions with a large number of independent variables [8]. It is especially suitable for such complicated problems as modelling of fracture networks, where there are many parameters and statistics to optimize. The algorithm brings about a number of modelling advantages compared to previous methods. First, it contains no mathematical assumptions on statistical distributions of fracture properties. Second, it offers more advanced data integration and simulation result. One of the remarkable advantages of global optimization is the flexibility of the objective function. Fracture data of different scales