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Reduction of uncertainty using sensitivity analysis methods for infinite random sets of indexable type

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ABSTRACT

In this paper we deal with the question "which is the best way to spend our resources in order to decrease the width of the interval $[\mathrm{Bel}(F), \mathrm{Pl}(F)]$ in Dempster–Shafer evidence theory?". A solution based on sensitivity analysis techniques using the Hartley-like measure of nonspecificity is proposed. This technique is a generalization of an approach introduced by Ferson and Tucker [S. Ferson, W.T. Tucker, Sensitivity in risk analysis with uncertain numbers, Report SAND2006-2801, Sandia National Laboratories, Albuquerque, NM, July 2006. <http://www.ramas.com/sensanal.pdf>; S. Ferson, W.T. Tucker, Sensitivity analysis using probability bounding, Reliability Engineering and System Safety 91 (10–11) (2006) 1435–1442], which does not require the calculation of the probability box associated to the output Dempster–Shafer structure after the application of the extension principle for random sets. The proposed technique is computationally much more efficient than the one of Ferson and Tucker by several orders of magnitude. Finally, the extension principle of Dubois and Prade [D. Dubois, H. Prade, Random sets and fuzzy interval analysis, Fuzzy Sets and Systems 42 (1) (1991) 87–101] is generalized for infinite random sets of indexable type.

1. Introduction

Given a mathematical model of a system, the main aim of sensitivity analysis is to analyze the influence of the model outputs with regard to the variation in the model inputs. If small changes of an input parameter result in relatively large changes in a model output, then that model is said to be *sensitive* to the parameter. This useful tool allows the analyst to recognize which input parameters affect the most the system response.

Sensitivity analysis has been widely employed in different fields of research like finance, optimization, optimal design, and control systems to assess the influence of the parameters on the state of the system and to gain insight into the model behavior (see e.g. [29, and references therein]). In particular, in reliability analysis of structural systems, sensitivity analysis is used to (a) measure how sensitive the probability of failure is to small changes in the material, load or geometry properties of the system, (b) recognize which are the design variables that have more influence in that variation, and to (c) explore the sensitivity to model assumptions as well as to the uncertainty in input variables. In consequence, sensitivity analysis is a fundamental complement to reliability analysis, because it provides an evaluation of the robustness of the design, while allowing to estimate how much less uncertainty in the computations we would have if additional knowledge about an input variable were available.

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A criticism against imprecise-probabilistic-based uncertainty techniques and in particular of random and evidence theories is that the interval $[\text{Bel}(F), \text{Pl}(F)]$ is too wide to make useful decisions. What these criticisms do not take into account is that these theories produce such wide bounds because they take into consideration all the available information, without including additional suppositions in the calculation that will be reflected in the results. However, those criticisms raise the natural question "which is the best way to spend our resources in order to reduce the width of the interval $[\text{Bel}(F), \text{Pl}(F)]$?". This question is not easy to answer because given the imprecise information that is available, it is not possible, in general, to identify a unique best solution to the problem. To do so, we will have to make anyway some suppositions in the calculation, by synthetically reducing the uncertainty in a basic variable or group of basic variables in order to simulate the additional gain of information; we will do this by means of a procedure that we will call pinching. Only then, we will get some idea on how to answer the posed question. This issue will be studied in the present article, and a solution based on techniques of sensitivity analysis will be proposed.

In this paper, we will follow the convention that aleatory uncertainty is associated with probability distributions and epistemic uncertainty is associated with imprecision and lack of knowledge. Note that this is by no means universally accepted. Bayesians, for example, insist that the epistemic uncertainty can be represented as well by probability distributions (see e.g. [27]). Since we do not want to reenter the debate about the representation of epistemic uncertainty, we state it up front now.

The plan of this paper is as follows: first, in Section 2, a brief state-of-the-art review of sensitivity analysis techniques in evidence theory will be presented; Section 3 gives an overview about the relation between random sets and nonspecificity; then in Section 4 a method of sensitivity analysis will be proposed based on the Hartley-like measure of nonspecificity, and which turns out to be a generalization of a strategy recently proposed by Ferson and Tucker [10]; a numerical example using our suggested method will be presented in Section 5. The article finishes with conclusions and some additional comments.

2. Sensitivity analysis in evidence and random set theories

Even though evidence and random set theories (see e.g. Refs. [21,19]) have been present since the middle of the 1970s, only recently, Oberkampf and Helton [26] pointed out the need of procedures to conduct sensitivity analysis. In fact, Helton et al. [18] acknowledged that although evidence theory has become a known tool within the risk and reliability assessment community, they are unaware of any attempts to develop procedures of sensitivity analysis within it. To the author's knowledge, there are only a few contributions to the problem in consideration; in the following lines they will be succinctly described.

In the framework of imprecise probabilities, Hall [15,16] proposed a method to extend the method of variance-based sensitivity analysis (see e.g. [29]) to imprecise probability theory. Additionally, Hall [16] proposed two other sensitivity measures. One is based on the calculation of the partial expected value of perfect information on the credal set, which is defined as the expected gain in utility as the consequence of learning the correct value of the implied basic variables. The other method is based on the calculation of the extreme values of expectation of the Kullback–Leibler entropy for all distributions contained in the credal set.

Bae et al. [5] considered two methodologies for sensitivity analysis for the plausibility measure: one with respect to the basic mass assignment of each focal element (which allows to see which expert's opinion is a major uncertainty propagation source) and the other with respect to the vector of system parameters. Their approach is based on finding $\partial \text{Pl}(F)/\partial a$ or $\partial \text{Pl}(F)/\partial m_i$, where a is one of the several parameters that define the characterization of the basic variables (for example, a could stand for a mean or variance of a cumulative distribution function (CDF) or the parameters that define a possibility distribution) and m_i is the basic mass assignment of any focal element; although this method provide us information on which parameters have more influence on the value of $\text{Pl}(F)$, it is not a useful tool to reduce the uncertainty $\text{Pl}(F) - \text{Bel}(F)$ inasmuch as it is not possible to freely modify those parameters to reduce the value of $\text{Pl}(F) - \text{Bel}(F)$.

In the framework of probability bounds analysis (PBA, see e.g. Ref. [12]), Ferson and Tucker [13,11] and Ferson et al. [10] showed that PBA is itself a global sensitivity analysis of a probabilistic calculation because it defines bounds of a CDF (by means of probability boxes) that represent the uncertainty about known input distributions and projects this uncertainty through the model to identify a neighborhood of possible answers (another probability box) in a way that guarantees that the resulting bounds will enclose completely the CDF of the output. They also proposed a "meta"-sensitivity analysis to determine which variables are the ones that have the largest influence on the variability of the working probability boxes. Since in Section 4 a generalization of this strategy will be proposed, this method will be explained in some detail in the following lines.

2.1. The method of Ferson and Tucker

This "meta"-sensitivity analysis is based on the idea of pinching one or several input basic variables towards precise CDFs or constants with the aim of hypothetically reducing the epistemic uncertainty, the aleatory uncertainty or both; thereafter, the response of the system is computed in the form of a probability box, and in a posterior step an indicator that measures the amount of uncertainty contained in the output probability box is calculated. Finally, this indicator is compared to the one calculated without pinching the input basic variables.