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RESEARCH PAPER

Sensitivity Analysis of Simulated Annealing for Continuous Network Design Problems

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Abstract: In this paper, parameters choices of simulated annealing for continuous network design problems are discussed. A bi-level programming model for continuous network design problem is introduced. Objective function of the upper level is defined as the sum of the total travel time on the network and the total investment costs of link capacity expansions. The lower level problem is the user equilibrium assignment model, which is solved by the Gradient projection algorithm. Sensitivity analysis method is the first time used to analyze and compare the influence of the different selection of parameters to the implementation of simulated annealing algorithm. Suggestions of parameter selection are also given. Analysis demonstrates that the efficiency and precision of these methods can be improved clearly with the proposed suggestions.

Key Words: continuous network design problem; bi-level programming model; simulated annealing; sensitivity analysis

1 Introduction

Continuous network design problem (CNDP) is concerned with the optimal capacity expansion of existing links in a given network by minimizing the total system cost as well as considering the route choice behavior of individual users. Due to multiple objectives for formulating CNDP, it was modeled as a bilevel programming problem, where the upper level is a nonlinear programming problem to minimize the system cost and the lower level is user equilibrium (UE) problem to assume drivers' route choice behavior[1]. In the transportation area, the bilevel programming models of CNDP was first proposed by Abdulaal and LeBlanc[2]. There are many researchers focusing on the studies of CNDP. Early detailed reviews and improved models can be found in Gao *et al*.[1,3–5].

How to design efficient algorithms for solving the CNDP formulated as the bilevel model is an important work, and fascinates practitioners' interest[6]. In this paper, we have focused on the algorithm implementation issues with simulated annealing to solve CNDP. Some early work on this topic can be referred to Friesz *et al*.[7,8]. Recently, both Gui[9] and Liu[10] studied the algorithm design problems of CNDP solved by simulated annealing, and the difference lies in using different algorithms for solving the UE on the lower problem. Xu *et al*.[11] compared the differences to solve CNDP with genetic algorithm and simulated annealing algorithm under different demand, and concluded that the implementation efficiency of the simulated annealing algorithm is higher than genetic algorithm under low demand, and it needs more operations with genetic algorithm to arrive at the same precision level.

When the simulated annealing algorithm is used to solve CNDP, the setting of parameters have clear effects on the solution and solving time, for example, the choice methods and choosing rulers of the inner iteration number, initial temperature, the temperature decrease rate, and the lowest temperature, and so on. It is a difficult problem to tell how to combine these parameters in order to get the best performance of the algorithm. Up to now, there is no theoretical method to approach this problem, and the setting of the parameters depends on experience or trial. In this paper, the parameter setting problems is first discussed with sensitivity analysis methods when using simulated annealing algorithm to solve CNDP, which could provide useful advice for the parameters setting.

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2 Bilevel programming formulation for CNDP

This section provides a summary of bilevel programming formulation for CNDP, and two sensitivity analysis methods including one-at-a-time designs (OATD) and factorial designs (FD). The presentation in this summary section follows Gao *et al.*[1,12]. Notation is provided firstly for convenience, followed by the bilevel programming formulation and OATD and FD methods.

2.1 Notation

Considering a transportation network $\boldsymbol{G}(\boldsymbol{N}, \boldsymbol{A})$, the following notations are given:

$\boldsymbol{A}$—set of links, $\forall a \in \boldsymbol{A}$;

$\boldsymbol{R}$—set of origins, $\forall r \in \boldsymbol{R}$;

$\boldsymbol{S}$—set of destinations, $\forall s \in \boldsymbol{S}$;

$\boldsymbol{P}_{rs}$—set of routes between OD pair (r, s), $\forall r \in \boldsymbol{R}$, $\forall s \in \boldsymbol{S}$;

x_a—flow on link $a, \forall a \in \boldsymbol{A}$, $\boldsymbol{x} = (\cdots, x_a, \cdots)$;

f_p^{rs}—flow on route p between OD pair (r, s), $p \in \boldsymbol{P}_{rs}$, $\boldsymbol{f} = (\cdots, f_p^{rs}, \cdots)$;

q_{rs}—travel demand between OD pair (r, s), $q = (q_{rs})_{|R| \times |S|}$;

$\delta_{a,p}^{rs}$—indicator variable: if link a is on route p between OD pair (r, s), $\delta_{a,p}^{rs} = 1$; $\delta_{a,p}^{rs} = 0$, otherwise. $(\Delta^{rs})_{a,p} = \delta_{a,p}^{rs}$, $\boldsymbol{\Delta} = (\cdots, \Delta^{rs}, \cdots)$;

y_a—capacity expansion on link $a \in \boldsymbol{A}$, $\boldsymbol{y} = (\cdots, y_a, \cdots)$, $y_a \in [\underline{y}_a, \bar{y}_a]$;

$G_a(y_a)$—investment costs function on link a, $\boldsymbol{G}(\boldsymbol{y}) = (\cdots, G_a(y_a), \cdots)$;

$t_a(x_a, y_a)$—travel time on link $a \in \boldsymbol{A}$, $\boldsymbol{t}(\boldsymbol{x}, \boldsymbol{y}) = (\cdots, t_a(x_a, y_a), \cdots)$;

ϕ—conversion factor from investment cost to travel cost.

2.2 Problem description

Thus CNDP can be formulated in terms of the bi-level programming model as follows[1]:

(P1) (L1)

$$\min z(\mathbf{x}, \mathbf{y}) = \sum_{a \in \mathbf{A}} \int_0^{x_a(\mathbf{y})} t_a(v, y_a)\, \mathrm{d}v \tag{1}$$

$$\text{s.t.} \quad \sum_{p \in \boldsymbol{P}_{rs}} f_p^{rs} = q_{rs}, \forall r \in \boldsymbol{R}, s \in \boldsymbol{S} \tag{2}$$

$$f_p^{rs} \geq 0, \forall r \in \boldsymbol{R}, s \in \boldsymbol{S}, p \in \boldsymbol{P}_{rs} \tag{3}$$

$$x_a = \sum_r \sum_s \sum_p f_p^{rs} \delta_{a,p}^{rs}, \forall a \in \boldsymbol{A} \tag{4}$$

(P1) (U1)

$$\min Z = \sum_{a \in \boldsymbol{A}} t_a(x_a(\boldsymbol{y}), y_a) x_a(\boldsymbol{y}) + \phi \sum_{a \in \boldsymbol{A}} G_a(y_a) \tag{5}$$

$$\text{s.t.} \quad \underline{y}_a \leq y_a \leq \bar{y}_a, \forall a \in \boldsymbol{A} \tag{6}$$

Equation (5) is the objective function of CNDP, which minimizes the whole network cost and investment cost by improving some links' ability. In this paper, the ϕ can set 1 or 1.5, and $G_a(y_a)=d_a \cdot y_a$ or $G_a(y_a)=d_a \cdot (y_a)^2$. Expression (6) requires the improved ability that is nonnegative and satisfies the constraints of upper and lower bounds.

3 Algorithm

In this paper, the gradient projection (GP) was used to solve the UE model (L1). GP is a typical path-based algorithm, and the convergence is faster than the Frank-Wolfe algorithm[13].

The overall process with simulated annealing algorithm to solve CNDP (P1) (U1) is summarized as follows:

Step 1: initialization. Given randomly the decision variable an initial solution y^0 in its field, and let $y = y^0$, it can get the value of $\boldsymbol{x}$ and corresponding objective function value $Z(x, y)$ by solving the lower UE model. Set the inner iteration number M, the initial temperature value T_0, the lowest temperature ε and set $T=T_0$, the temperature decrease rate α, and initial stepsize l.

Step 2: set the inner iteration number k=0. And set $\hat{\boldsymbol{y}} = \boldsymbol{y} + l \cdot \boldsymbol{U}$, where $\boldsymbol{U}$ is a stochastic vector, $\boldsymbol{U} = (\cdots, U_i, \cdots)$, and U_i is a random variable and follows the uniform distribution on [–1, 1]. Solve the lower UE model, get the value $\hat{\boldsymbol{x}}$, and calculate the objective function value $Z(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}})$.

Step 3: if $\Delta Z = Z(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}) - Z(\boldsymbol{x}, \boldsymbol{y}) < 0$, let $\boldsymbol{y} = \hat{\boldsymbol{y}}$; or get the value $\hat{\boldsymbol{y}}$ with the possibility $p = \exp[(-\Delta Z)/T]$, that is, generate a random value r on [0, 1], set $\boldsymbol{y} = \hat{\boldsymbol{y}}$ if $r<p$.

Step 4: If $k=M$, go to Step 5, otherwise, set $k=k+1$ and return to Step 2.

Step 5: If $T<\varepsilon$, algorithm stop and output decision variable y and objective function value Z; otherwise, update stepsize l, and current temperature $T=\alpha \cdot T$, and return Step 2.

In the above algorithm, the initial stepsize l depends on the practical problem, and the updating of stepsize can depend on $l=l \cdot \beta$, where β is the stepsize decrease ratio. Assume l_0 is the maximization stepsize, and l_f is the minimization stepsize, it can set $\beta = \sqrt[n]{l_g / l_0}$, where $n=(\ln\varepsilon - \ln T_0)/\ln\alpha$, this method can reduce the number of the parameters. Furthermore, to avoid the small changes of decision variable y to solve the lower model (which add the cost of implementation time), it is set that passing over to solve the lower problem with the small changes of y.

4 Sensitivity analysis

In the contexts of numerical modeling, sensitivity analysis (SA) study the relationships between information flowing in and out of model. SA is widely used in model development, verification, calibration, model identification and mechanism reduction. It can assist the modeler to determine whether the parameters are sufficiently precise for the model to give reliable predictions. In this paper, the sensitivity analysis (SA) is introduced to appraise the estimated parameters of simulated annealing to the CNDP. Two methods (one-at-a-time designs (OATD) and factorial designs (FD)) are used to analyze the effects of parameters[12].