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Finite element response sensitivity analysis of multi-yield-surface J_2 plasticity model by direct differentiation method

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ABSTRACT

Finite element (FE) response sensitivity analysis is an essential tool for gradient-based optimization methods used in various sub-fields of civil engineering such as structural optimization, reliability analysis, system identification, and finite element model updating. Furthermore, stand-alone sensitivity analysis is invaluable for gaining insight into the effects and relative importance of various system and loading parameters on system response. The direct differentiation method (DDM) is a general, accurate and efficient method to compute FE response sensitivities to FE model parameters. In this paper, the DDM-based response sensitivity analysis methodology is applied to a pressure independent multi-yield-surface J_2 plasticity material model, which has been used extensively to simulate the nonlinear undrained shear behavior of cohesive soils subjected to static and dynamic loading conditions. The complete derivation of the DDM-based response sensitivity algorithm is presented. This algorithm is implemented in a general-purpose nonlinear finite element analysis program. The work presented in this paper extends significantly the framework of DDM-based response sensitivity analysis, since it enables numerous applications involving the use of the multi-yield-surface J_2 plasticity material model. The new algorithm and its software implementation are validated through two application examples, in which DDM-based response sensitivities are compared with their counterparts obtained using forward finite difference (FFD) analysis. The normalized response sensitivity analysis results are then used to measure the relative importance of the soil constitutive parameters on the system response.

1. Introduction

Finite element (FE) response sensitivities represent an essential ingredient for gradient-based optimization methods required in various sub-fields of structural and geotechnical engineering such as structural optimization, reliability analysis, system identification, and FE model updating [1,2]. In addition, FE response sensitivities are invaluable for gaining insight into the effects and relative importance of system and loading parameters in regards to system response.

Several methods are available for response sensitivity computation, including the finite difference method (FDM), the adjoint method (AM), the perturbation method (PM), and the direct differentiation method (DDM). These methods are described by Zhang and Der Kiureghian [3], Kleiber et al. [2], Conte et al. [4–6], Gu and Conte [7], Scott et al. [8], and Haukaas and Der Kiureghian [9]. The FDM is the simplest method for response sensitivity computation, but is computationally expensive and can be negatively affected by numerical noise (i.e., truncation and round-off errors). The AM is extremely efficient for linear and nonlinear elastic systems, but is not a competitive method for path-dependent (i.e., inelastic) problems. The PM is computationally efficient, but generally not very accurate. The DDM, on the other hand, is general, accurate and efficient and is applicable to any material constitutive model (both path-independent and path-dependent). The computation of FE response sensitivities to system and loading parameters based on the DDM requires extension of the FE algorithms for response-only computation [5].

Based on DDM, this paper presents a derivation of response sensitivities with respect to material parameters of an existing material model, the multi-yield-surface J_2 plasticity model. This model was first developed by Iwan [10] and Mroz [11], then applied by Prevost [12–14] to soil mechanics. It was later modified and implemented in OpenSees [15–17] by Yang [18] and Elgamal et al. [19]. OpenSees is an open source software framework for advanced modeling and simulation of structural and geotechnical systems developed under the auspice of the Pacific Earthquake Engineering Research (PEER) Center. In contrast to the classical J_2 (or Von Mises) elasto-plastic behavior with a single yield surface,

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multi-yield-surface J_2 plasticity employs the concept of a field of plastic moduli [12–14] to achieve a piecewise linear elasto-plastic behavior under cyclic loading conditions. This field is defined by a collection of nested yield surfaces of constant size (i.e., no isotropic hardening) in the stress space, which define the regions of constant plastic shear moduli (and therefore constant tangent shear moduli). The stress sensitivity to material parameters is computed by differentiating consistently the constitutive law integration algorithm, adding the contributions from all yield surfaces that affect the stress computation at the current time step.

The existing implementation in OpenSees [15] of the multi-yield-surface J_2 plasticity model [18,19] considered is then extended to enable response sensitivity computation using the DDM-based algorithm developed in this paper. The DDM-based algorithm was implemented in OpenSees by extending the existing framework for sensitivity and reliability analysis developed by Der Kiureghian et al. [20], Haukaas and Der Kiureghian [21], and Scott and Haukaas [22].

The work presented in this paper extends significantly the framework of DDM-based response sensitivity analysis, since it enables numerous applications involving the use of the multi-yield-surface J_2 plasticity material model. Although this material model is a rather old model, it remains an effective and robust model to simulate the undrained response of cohesive materials under cyclic and seismic loading conditions [12–14,16,18,19,23–26]. Also, it is operational in OpenSees [18] through which soil–structure-interaction studies may be conducted by a large user community. Thus, an area of application of the present DDM-based FE response sensitivity analysis scheme is in earthquake loading (undrained) for geotechnical cohesive soils, with applications to soil-foundation-structure interaction scenarios [16,17]. Response sensitivity analysis results are needed as input for reliability, optimization, and FE model updating applications. Therefore, the contribution of this paper potentially improves significantly the computational efficiency of such applications to a wide class of geotechnical systems [27–29] and soil-foundation-structure interaction systems involving the dynamic undrained shear response of cohesive soils.

The developments presented in this paper include new implementation details of the DDM that can carry over to other advanced constitutive models. (1) To the authors' knowledge, in past work, the DDM-based response sensitivity analysis methodology has been implemented for uniaxial material constitutive models [2,5,6,8] and three-dimensional (3D) single surface J_2 plasticity models [2,3] with implicit constitutive law integration schemes. In this paper, the DDM methodology is extended to a general 3D elasto-plastic material constitutive model, in which the multi-yield-surface J_2 plasticity approach is utilized. (2) In this plasticity model, the stress state at the current load/time step is obtained through an explicit corrective iteration scheme, which accumulates contributions from all yield surfaces involved, the number of which varies from load/time step to load/ time step [30]. The DDM-based response sensitivity algorithm follows exactly the corrective iteration process for stress computation. (3) The computation of the DDM-based FE response sensitivity requires the consistent and not the continuum tangent material moduli [5]. The consistent tangent moduli consist of an unsymmetrical fourth-order tensor (exhibiting only minor symmetries, $D_{ijkl} = D_{jikl} = D_{jilk} = D_{jilk}$, but $D_{ijkl} \neq D_{klij}$). They are computed by differentiating the stress tensor with respect to the strain tensor by following exactly the stress computation algorithm as presented in [30]. (4) The sensitivities of the kinematic hardening parameters defining the initial configuration of the multi-yield surfaces are required at the initiation of the response and response sensitivity computation. Furthermore, the sensitivities of the kinematic hardening parameters defining the active and inner yield surfaces must be updated at each load/time step.

Two application examples are provided to validate the new response sensitivity algorithm and its implementation using the finite difference method (FDM). As an application of response sensitivity analysis, the response sensitivity results are used to measure the relative importance of the soil material parameters of different soil layers on the displacement response of the soil.

2. Constitutive formulation of multi-yield-surface J_2 plasticity model and numerical integration

2.1. Multi-yield surfaces

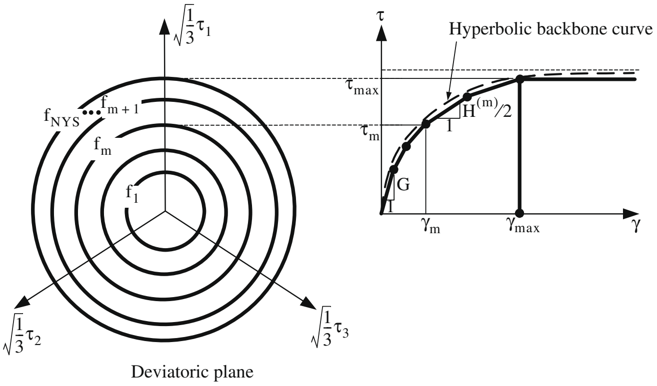
Each yield surface of this multi-yield-surface J_2 plasticity model is defined in the deviatoric stress space as [23]

$$f = \left\{\frac{3}{2}((\boldsymbol{\tau} - \boldsymbol{\alpha}) : (\boldsymbol{\tau} - \boldsymbol{\alpha}))\right\}^{\frac{1}{2}} - K = 0, \tag{1}$$

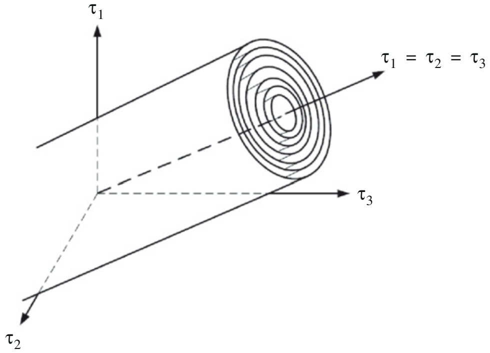
where $\boldsymbol{\tau}$ denotes the deviatoric stress tensor and $\boldsymbol{\alpha}$, referred to as back-stress tensor, denotes the center of the yield surface $\{f = 0\}$ in the deviatoric stress space. Parameter K represents the size ($\sqrt{3/2}$ times the radius) of the yield surface which defines the region of constant plastic shear modulus. The dyadic tensor product of tensors $\mathbf{A}$ and $\mathbf{B}$ is defined as $\mathbf{A}:\mathbf{B} = A_{ij}B_{ij}$ $(i, j = 1, 2, 3)$. The back-stress $\boldsymbol{\alpha}$ is initialized to zero at the start of loading.

In geotechnical engineering, the nonlinear shear behavior of soil materials is described by a shear stress–strain backbone curve [18,19] as shown in Fig. 1a. The experimentally determined backbone curve can be approximated by the hyperbolic formula [31] as

$$\tau = \frac{G\gamma}{1 + (\gamma/\gamma_r)}, \tag{2}$$



(a) Octahedral shear stress-strain (after [12])



(b) Von Mises multi-yield surfaces

Fig. 1. Yield surfaces of multi-yield-surface J_2 plasticity model in principal deviatoric stress space.