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Fractal finite element method based shape sensitivity analysis of multiple crack system

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ABSTRACT

This paper presents fractal finite element based continuum shape sensitivity analysis for a multiple crack system in a homogeneous, isotropic, and two dimensional linear-elastic body subjected to mixed-mode (modes I and II) loading conditions. The salient feature of this method is that the stress intensity factors and their derivatives for the multiple crack system can be obtained efficiently since it only requires an evaluation of the same set of fractal finite element matrix equations with a different fictitious load. Three numerical examples are presented to calculate the first-order derivative of the stress intensity factors or energy release rates.

1. Introduction

Methods based on fractal geometry concepts such as fractal finite element method (FFEM), to generate infinite number of finite elements around the crack tip to capture the crack tip singularity have been successfully adopted to solve many kinds of crack problems under mode-I and mixed mode loading conditions [1–8]. Compared with other numerical methods like finite element method (FEM), FFEM has several advantages. First, by using the concept of fractal geometry, infinite finite elements are generated virtually around the crack tip, and hence the effort for data preparation can be minimized. Second, based on the eigenfunction expansion of the displacement fields [9,10], the infinite finite elements that generate virtually by fractal geometry around the crack tip are transformed in an expeditious manner. This results in reducing the computational time and the memory requirement for fracture analysis of cracked structures. Third, no special finite elements and post-processing are needed to determine the stress intensity factors (SIFs) and their derivatives. Finally, as the analytical solution is embodied in the transformation, the accuracy of the predicted SIFs and their derivatives is high.

In addition to the SIFs, the derivatives of the SIFs are often required for reliable analysis of crack growth behavior under LEFM conditions. Hence, sensitivity analysis of a crack-driving force plays an important role in many fracture mechanics applications which includes the prediction of stability and arrest of a single crack [11], the growth pattern analysis of a system of interacting cracks [12,13], configurational stability analysis of evolving cracks [14], size effect model [15], stability analysis of crack path [16] and probabilistic fracture mechanics analysis [17,18]. Hwang et al. [19,20] clearly outlined the potential applications of the derivatives of the energy release rate and the SIFs. The first- and second-order reliability methods [21], frequently used in probabilistic fracture mechanics [22–24], require the gradient and Hessian of the performance function with respect to random parameters. In linear-elastic fracture mechanics (LEFM), the performance function is built on SIFs. Hence, both first- and/or second-order derivatives of the SIFs or energy release rates are needed for probabilistic

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Nomenclature

a	crack length
$\boldsymbol{a}$	William's eigenfunction series coefficients $\{a_0^I, a_0^{II}, a_1^I, a_1^{II}, a_2^I, a_2^{II}, \ldots\}^T$
$\dot{\boldsymbol{a}}^p$	Material derivative of $\boldsymbol{a}$ due to the perturbation of the structural domain shape around the pth crack tip
$a'_{\boldsymbol{V}^p}(\bullet,\bullet)$	structural fictitious load form
$a_\Omega(\bullet,\bullet)$	structural energy form
$\boldsymbol{B}$	strain displacement matrix
$\boldsymbol{d}$	nodal displacement vector
$\boldsymbol{d}_m$	displacement vector of the master nodes
$\boldsymbol{d}_r$	displacement vector of the nodes in the regular region other than the master nodes
$\boldsymbol{d}_s^{k\text{-}th}$	nodal displacement vector for the second and subsequent layers in the singular region
$\boldsymbol{d}_s^{1st}$	displacement vector of the slave nodes in the first layer of the singular region
$\dot{\boldsymbol{d}}^p$	material derivative of nodal displacement vector due to the perturbation of the structural domain shape around the pth crack tip
$\dot{\boldsymbol{d}}_m^p$	material derivative of displacements of the master nodes due to the perturbation of the structural domain shape around the pth crack tip
$\dot{\boldsymbol{d}}_r^p$	material derivative of displacements of the nodes in the regular region other than the master nodes due to the perturbation of the structural domain shape around the pth crack tip
$\dot{\boldsymbol{d}}_s^{1st^p}$	material derivative of displacements of the slave nodes in the first layer of the singular region due to the perturbation of the structural domain shape around the pth crack tip
$\boldsymbol{D}$	constitutive tensor
E	Young's modulus
$\boldsymbol{f}$	nodal force vector
$\boldsymbol{f}_s^{k\text{-}th}$	nodal force vector for the second and subsequent layers in the singular region
$\boldsymbol{f}_r^R, \boldsymbol{f}_m^R$	partitioned force vector in the regular region with respect to the nodes other than the master nodes and the master nodes
$\boldsymbol{f}_m^{1st}, \boldsymbol{f}_s^{1st}$	partitioned force vector for the first layer in the singular region with respect to the master nodes and the slave nodes
$\boldsymbol{f}'^p$	global force sensitivity vector due to the perturbation of the structural domain shape around the pth crack tip
$\boldsymbol{f}_s'^{k-\text{th}^p}$	nodal force sensitivity vector for the second and subsequent layers in the singular region due to the perturbation of the structural domain shape around the pth crack tip
$\boldsymbol{f}_r'^{R^p}, \boldsymbol{f}_m'^{R^p}$	partitioned force sensitivity vectors in the regular region with respect to the nodes other than the master nodes and the master nodes
$\boldsymbol{f}_m'^{1st^p}, \boldsymbol{f}_s'^{1st^p}$	partitioned force sensitivity vectors for the first layer in the singular region with respect to the master nodes and the slave nodes
$\boldsymbol{f}_I'^p$	element-level force sensitivity vector due to the perturbation of the structural domain shape around the pth crack tip
G_I	mode-I energy release rate
G_{II}	mode-II energy release rate
K_I	mode-I stress intensity factor
K_{II}	mode-II stress intensity factor
$\partial K_I/\partial a$	sensitivity of mode-I stress intensity factor
$\partial K_{II}/\partial a$	sensitivity of mode-II stress intensity factor
$\boldsymbol{k}_{IJ}'^p$	element-level stiffness sensitivity matrix due to the perturbation of the structural domain shape around the pth crack tip
$\boldsymbol{K}$	stiffness matrix
$\boldsymbol{K}_s^{k-th}$	stiffness matrix for the second and subsequent layers in the singular region
$\boldsymbol{K}_{rr}^R, \boldsymbol{K}_{rm}^R, \boldsymbol{K}_{mr}^R, \boldsymbol{K}_{mm}^R$	partitioned stiffness matrices in the regular region with respect to the nodes other than the master nodes and the master nodes
$\boldsymbol{K}_{mm}^{1st}, \boldsymbol{K}_{ms}^{1st}, \boldsymbol{K}_{sm}^{1st}, \boldsymbol{K}_{ss}^{1st}$	partitioned stiffness matrices for the first layer in the singular region with respect to the master nodes and the slave nodes
$\boldsymbol{K}'^p$	global stiffness sensitivity matrix due to the perturbation of the structural domain shape around the pth crack tip
$\boldsymbol{K}_s'^{k-th^p}$	stiffness sensitivity matrix for the second and subsequent layers in the singular region due to the perturbation of the structural domain shape around the pth crack tip
$\boldsymbol{K}_{rr}'^{R^p}, \boldsymbol{K}_{rm}'^{R^p}, \boldsymbol{K}_{mr}'^{R^p}, \boldsymbol{K}_{mm}'^{R^p}$	partitioned stiffness matrices in the regular region with respect to the nodes other than the master nodes and the master nodes
$\boldsymbol{K}_{mm}'^{1st^p}, \overline{\boldsymbol{K}}_{ms}'^{1st^p}, \overline{\boldsymbol{K}}_{sm}'^{1st^p}, \overline{\boldsymbol{K}}_{ss}'^{1st^p}$	partitioned stiffness matrices for the first layer in the singular region with respect to the master nodes and the slave nodes
$\ell_\Omega(\bullet)$	load linear form