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Sensitivity analysis by the adjoint variable method for optimization of the die compaction process in particulate materials processing

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ABSTRACT

Nonuniform shrinkage during sintering results from the nonuniform green density distribution in a compacted powder body and this creates a severe problem in net-shape forming. Therefore, proper selection of the process parameters, such as compaction die design, upper and lower punch displacements, and sequential tool motion, is very important for the optimum design of die compaction processing. Previously, these were guess-estimated based on experience. As a first step in developing a more scientific design technique in powder metallurgy, the equations for the adjoint variable method (AVM) for the non-steady-state uniaxial powder compaction process are derived in detail. The accuracy of the AVM is verified through the comparison of the design sensitivity with calculations performed using the finite difference method.

1. Introduction

Powder metallurgy and particulate materials (P/M^2) processes obtain the designed shape and mechanical strength using a two-step approach that first relies on uniaxial die compaction followed by sintering. Such an approach is applied to metals, polymers, ceramics, cermets, and mixtures of metal, ceramic, or carbide powders. Often a polymer or other lubricant is added to the powders to improve tool life. Since the range of sintered components is broad in terms of materials and shapes, the options in P/M^2 have rapidly developed to include many options.

In recent years, sintered products have been broadly adopted by the automotive industry for the purpose of reducing cost and weight [1]. Recently, other forming processes, such as powder injection molding and extrusion, have been developed for shaping the desired geometry [2], but uniaxial die compaction is still the most popular forming approach. After compaction, the sintered compact often distorts since the sintering dimensional change depends on green density. The nonuniform density distribution resulting from the compaction event leads to distortion and an inability to hold tight tolerances. There are two main causes for the nonuniform density: (1) the friction between the particles and tool surface and (2) nonuniform stress on the compaction body results from the pressure decay with depth in the green body. Therefore, often expensive finishing steps, such as machining, are needed to achieve the desired tolerances.

To reduce green density gradients in the compact, field engineers have mainly focused on reducing the friction between powders and tooling using lubricants. However, the nonuniform green density cannot be eliminated simply by adjusting the powder-tooling friction. In this paper, we studied a systematic methodology for determining the compaction process parameters, including upper and lower punch speed control as well as optimized die shape.

Previously, diverse optimization techniques have been used for forming processes such as extrusion, rolling, forging, and powder forging. They include backward tracing schemes [3,4], genetic algorithms [5,6], and derivative-based approaches [7–16]. Among these techniques, the derivative-based approaches are generally superior when considering the quality of the design and time consumption. In using derivative-based approaches, calculation of the design sensitivity (DS) is very important. To calculate the DS, analytical methods such as the adjoint variable method (AVM) and direct differentiation method (DDM) and numerical methods such as finite difference method (FDM) are being used. Since the FDM requires additional finite element (FE) calculations depending on the number of design variables (DV), it is not used in the optimization iterations. Rather it is used for verifying the DS by the analytical method.

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According to the method of treating the derivative of the FE solutions, such as velocity field with respect to design parameters, the analytical methods can be classified into the DDM and AVM. In the DDM the derivative is directly calculated but in the AVM it is treated by introducing the Lagrangian multipliers on the FE equations. In order to calculate the design sensitivity, that is the derivative of objective function with respect to DVs, it is necessary to calculate the derivative of the FE solution such as the velocity field, with respect to DVs. The calculation time for this derivative costs much and is proportional to the number of DVs. On the contrary to DDM, AVM does not directly calculate the derivative by introducing adjoint variable. Therefore, AVM is more powerful for calculating design sensitivity in terms of calculating time especially when the number of DVs is large [17]. However, the treatment of adjoint variable is very complicated and hard to derive especially in non-steady forming. Therefore, the AVM has been used in steady-state forming, while the DDM has been used in non-steady forming. However, Chung et al. [18] derived the equations for the AVM to find the optimal intermediate die shape in a non-steady-state forging process; this gave an application of the AVM to non-steady forming process. In this study the AVM is adapted to reduce time cost through the derivation of the equations for the AVM in non-steady forming of porous material. See Ref. [19] for the detail of design sensitivity analysis.

For the first step of developing the optimization tool, this paper considers the uniaxial die compaction process. Since die compaction is carried out in room temperature, the material properties depend on just the relative density and effective strain. Therefore, consideration of the derivatives related to relative density and effective strain is included in the derivation of the AVM. In Section 2 of this paper, the constitutive model for a powder body in die compaction process is given and the boundary value problem (BVP) and FE formulation are explained. In Section 3, equations for the AVM in die compaction process are generally derived using FE formulation to generate a general form of the objective function. In Section 4, the optimization scheme based on the DS is introduced. In Section 5, the DS obtained by the AVM is verified by comparing it with that by the FDM. In Section 6, the upper and lower punch speed is optimized to obtain uniform relative density distribution. Finally, in Section 7, we offer conclusions and outline further work.

2. Finite element (FE) formulation

2.1. Yield criteria of porous body

In the die compaction process, the deformation behavior of the powder is based on a yield criterion. Unlike bulk solids, the volume change in compaction requires the yield criterion for a powder including the hydrostatic pressure as follows:

$$AJ_2' + BJ_1^2 = \bar{\sigma}^2, \tag{1}$$

where A and B are material parameters that are functions of relative density, J_1 is the first invariants of the stress tensor σ_{ij}, J_2' is the second invariant of the deviatoric part of stress tensor σ_{ij}, and $\bar{\sigma}$ is the effective stress of powder continuum. The effective stress of powder continuum can be expressed by the function of the effective stress on the base material and relative density as follows.

$$\bar{\sigma} = \sqrt{\beta(\rho)}\bar{\sigma}_m(\bar{\varepsilon}_m, \dot{\bar{\varepsilon}}_m, T), \tag{2}$$

where β is the material parameter with function of relative density ρ, $\bar{\varepsilon}_m$ the effective strain of the base material, $\dot{\bar{\varepsilon}}_m$ the effective strain rate of base material, and T the temperature.

From the uniaxial tension or compression test, the relation of A and B is $A = 3(1-B)$. According to the definition of A and β, many criteria have been suggested [18,20–23]. We have rich experiences of Shima and Oyane's [21] criterion in die compaction process [24–26]. Therefore, in this study, Shima and Oyane's criterion is used as follows:

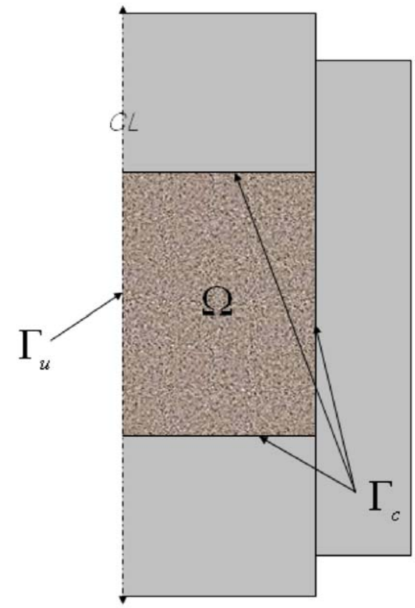

Fig. 1. Diagram for notation in domain and boundary conditions.

$$A = \frac{3}{1+0.694(1-\rho)} \text{ and } \beta = \frac{\rho^5}{1+0.694(1-\rho)}. \tag{3}$$

2.2. Boundary value problem (BVP)

Consider a deforming body Ω with the velocity $u_i = \bar{u}_i$ prescribed on a part Γ_u. Let Γ_c be the remainder of the surface and assume that represents the tool–workpiece interface, as shown in Fig. 1. The boundary value problem associated with current moment in the non-steady-state plastic deformation process can be given as follows:

Find a velocity field u_i satisfying

- mass balance:

$$\frac{\dot{\rho}}{\rho} = -\dot{\varepsilon}_v, \tag{4}$$

  where ρ is time derivative of relative density ρ and $\dot{\varepsilon}_v$ is the first invariant of strain tensor $\dot{\varepsilon}_{ij}$,
- equilibrium equation:

$$\sigma_{ij,j} + f_i = 0, \tag{5}$$

  where $\sigma_{ij,j}$ is the gradient of stress tensor σ_{ij} and f_i is the external body force,
- constitutive equation:

$$\sigma_{ij}' = \frac{2}{A}\frac{\bar{\sigma}}{\dot{\bar{\varepsilon}}}\dot{\varepsilon}_{ij}', \tag{6}$$

$$p = -\frac{1}{3(3-A)}\frac{\bar{\sigma}}{\dot{\bar{\varepsilon}}}\dot{\varepsilon}_v, \tag{7}$$

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{A}\dot{\varepsilon}_{ij}'\dot{\varepsilon}_{ij}' + \frac{1}{3(3-A)}\dot{\varepsilon}_v^2}, \tag{8}$$