



## Coupled vibration and parameter sensitivity analysis of rocking-mass vibrating gyroscopes

M. Ansari<sup>a</sup>, E. Esmailzadeh<sup>b,\*</sup>, N. Jalili<sup>c</sup>

<sup>a</sup> Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

<sup>b</sup> Faculty of Engineering and Applied Science, University of Ontario Institute of Technology Oshawa, Ontario, Canada L1H 7K4

<sup>c</sup> Smart Structures and NEMS Laboratory, Department of Mechanical Engineering, Clemson University, Clemson, SC 29634-0921, USA

### ARTICLE INFO

#### Article history:

Received 3 February 2009

Received in revised form

15 June 2009

Accepted 22 June 2009

Handling Editor: M.P. Cartmell

Available online 14 July 2009

### ABSTRACT

Vibrating beam gyroscopes are widely used to measure the angle or the rate of rotation of many mechanical systems. The vibration and parameters sensitivity analyses of a specific type of vibrating beam gyroscope namely rocking-mass gyroscopes are presented in this paper. These types of gyroscopes by far have a better performance than the conventional single-beam gyroscopes. The system comprises of four slender beams attached to a rigid substrate, undergoing coupled flexural and torsional vibrations with a finite mass attached in the middle. Two of the beams carry piezoelectric patch actuators on top, while the other two possess piezoelectric patch sensors. Using extended Hamilton's principle, the resulting eight coupled partial differential equations of motion with their corresponding boundary conditions are derived. In spite of the need for a high computational power, the system is analysed in the frequency domain using an exact method and the closed-form characteristic equations for two cases of fixed and rotating base support are obtained. Furthermore, a detailed parameter sensitivity analysis is carried out to determine the effects of different parameters on the complex natural frequencies of the system. Results presented are valuable in the design of this type of gyroscope as the exact resonant conditions and the sensitivity of the system parameters play important roles in the dynamic performance of gyroscopes.

© 2009 Elsevier Ltd. All rights reserved.

### 1. Introduction

Due to the wide range of the applications of the vibrating mass gyroscopes; they are being used in many navigational applications, namely, aerospace, marine and automobile industries. Hence, the detailed study of such systems has always been of great interest to engineers and researchers.

In most of these types of gyroscopes, the bending and torsional vibrations are coupled. The theory of coupled flexural-torsional vibrations for thin-walled beams was first developed by Timoshenko and Young [1]. The free flexural/torsional vibration of an Euler-Bernoulli beam with a rigid tip mass was studied by Oguamana [2]. He presented explicit expressions for the frequency equation, mode shapes and their orthogonality relationship and investigated the effects of different parameters on the fundamental frequencies of the system. Salarieh and Gorashi [3] continued his work, but used the Timoshenko beam theory. They studied the effects of the shear deformation and the rotary inertia on the free vibration response of a Timoshenko beam with a rigid tip mass. Gokdag and Kopmaz [4] extended the work of

\* Corresponding author. Tel.: 1905 721 8668x2600; fax: 1905 721 3370.

E-mail address: [ezadeh@uoit.ca](mailto:ezadeh@uoit.ca) (E. Esmailzadeh).

Nomenclature			
$b$	width of the beams	$t_p$	piezoelectric layers thickness
$b_M$	rocking-mass width	$T_{1b}$	translational kinetic energy of the four beams due to the translational motion
$C_{ij}, C'_{ij}$	elements of matrices	$T_{1M}$	translational kinetic energy of the rocking-mass due to the translational motion
$d_{31}$	piezoelectric constant of actuators	$T_{2b}$	translational kinetic energy of the four beams due to the rotational motion
$E$	elastic modulus of beams	$T_{2M}$	translational kinetic energy of the rocking-mass due to the rotational motion
$E_p$	elastic modulus of actuators	$V$	total potential energy of the system
$El_b$	flexural rigidity of the beams	$\nu(t)$	voltage applied to actuators
$El_p$	flexural rigidity of the actuator	$w_i$	bending of $i$ th beam
$G$	beams shear modulus	$W_{nc}$	total non-conservative work done on the system
$GJ_b$	torsional rigidity of the beams	$\theta_i$	torsion of $i$ th beam
$GJ_p$	torsional rigidity of the actuator	$\rho_b$	mass per unit length of the beams
$h_M$	Rocking-mass height	$\rho_p$	mass per unit length of the piezoelectric actuator
$i$	$\sqrt{-1}$	$\psi_i, \theta_i$	angle of rotations of $i$ th beam
$I_{xb}$	mass moment of inertia of the beams	$\Omega_1$	rotation about $X_i$ -axis
$I_{xp}$	mass moment of inertia of the piezoelectric actuator	$\Omega_2$	rotation about $Y_i$ -axis
$l$	length of rocking-mass	$\Omega_3$	rotation about $Z_i$ -axis
$l_1$	start position of piezoelectric actuator	$\omega$	vibration frequency
$l_2$	end position of piezoelectric actuator	$\omega_{ix}$	angular velocity component of $i$ th beam in $x$ -direction
$L_i$	length of $i$ th beam	$\omega_{iy}$	angular velocity component of $i$ th beam in $y$ -direction
$M$	rocking-mass	$\omega_{iz}$	angular velocity component of $i$ th beam in $z$ -direction
$M_p$	piezoelectric actuator control moment		
$Q_n(x)$	amplitude of torsion		
$P_n(x)$	amplitude of bending		
$s_1$	position of point P on beam 1		
$T$	total kinetic energy of the system		
$t_b$	beams thickness		

Oguamana [2] by studying the coupled flexural/torsional vibrations of a beam with either the tip or in the span mass attachments.

In a series of studies, Jalili and his research team [5–7] worked on the vibrating gyroscopic systems experiencing coupled flexural/torsional vibrations. Their first work was to develop a thorough modeling framework for vibrating gyroscopes subjected to general support motion by considering both the flexural and torsional vibrations [5]. In a subsequent work, they include a novel piezoelectric actuation for the vibrating beam gyroscope, which was modeled as an Euler-Bernoulli beam with a tip load subjected to the base rotation. They investigated the effect of the cross-axis in single beam vibratory gyroscopes [6], and also the influence of the substrate motion on the performance of the ring microgyroscopes [7].

Although vibrating beam gyroscopes are becoming the most widely used gyroscopes in many applications [6], but they possess a very important drawback, which produces the cross-coupling error in the measurements [6,8]. The vibrating beam gyroscope is typically used to measure the rotational rate around one of the axes. In practice, however, there are always some secondary rotations present in the system. These secondary base rotations can produce significant errors in the measurement of the gyroscope output (cross-axis error). The gyroscopic output increases significantly even for a small secondary rotation. This increased output could be interpreted as the gyroscope output due to the primary base rotation and can hence, develop errors in the measurement [6].

In spite of single beam gyroscopes, the rocking-mass gyroscope does not have those drawbacks, and can accurately measure the rate of rotation. Due to the complexities involved in the modelling and performance analysis of this kind of gyroscope, only few studies have been carried out in this area. The fabrication and design of a rocking-mass gyroscope was studied by Tang and Gutierrez [9], but the operating principle of the device was not fully discussed. Royle and Fox [10] presented an analysis of the mechanics of an oscillatory rate gyroscope that is actuated and sensed using thin piezoelectric actuators and sensors. A modeling framework for these systems, which forms the basis of this paper, is an extension to the work reported by Bhadbhade [11].

The present research undertakes the vibration analysis of a rocking-mass gyroscope, which comprises of a rotating rigid substrate and an assembly of four cantilever beams with a rigid mass attached to them in the middle, as shown in Fig. 1. The objective of the research is to develop a detailed mathematical modeling of the system. The governing equations of motion, using the extended Hamilton's principle, are derived. Since the closed-form solutions can serve as the benchmarks for validating the results obtained from either the numerical calculations or experimental results, the closed-form equations are developed for the frequency characteristic equations of the system for either a fixed supporting base or a rotating one. These exact equations are very important and useful, since their solutions would not only provide exact

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات