Journal of Sound and Vibration 327 (2009) 564–583

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Coupled vibration and parameter sensitivity analysis of rocking-mass vibrating gyroscopes

M. Ansari [a], E. Esmailzadeh [b,*], N. Jalili [c]

[a] Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1
[b] Faculty of Engineering and Applied Science, University of Ontario Institute of Technology Oshawa, Ontario, Canada L1 H 7K4
[c] Smart Structures and NEMS Laboratory, Department of Mechanical Engineering, Clemson University, Clemson, SC 29634-0921, USA

ARTICLE INFO

Article history:
Received 3 February 2009
Received in revised form
15 June 2009
Accepted 22 June 2009
Handling Editor: M.P. Cartmell
Available online 14 July 2009

ABSTRACT

Vibrating beam gyroscopes are widely used to measure the angle or the rate of rotation of many mechanical systems. The vibration and parameters sensitivity analyses of a specific type of vibrating beam gyroscope namely rocking-mass gyroscopes are presented in this paper. These types of gyroscopes by far have a better performance than the conventional single-beam gyroscopes. The system comprises of four slender beams attached to a rigid substrate, undergoing coupled flexural and torsional vibrations with a finite mass attached in the middle. Two of the beams carry piezoelectric patch actuators on top, while the other two possess piezoelectric patch sensors. Using extended Hamilton's principle, the resulting eight coupled partial differential equations of motion with their corresponding boundary conditions are derived. In spite of the need for a high computational power, the system is analysed in the frequency domain using an exact method and the closed-form characteristic equations for two cases of fixed and rotating base support are obtained. Furthermore, a detailed parameter sensitivity analysis is carried out to determine the effects of different parameters on the complex natural frequencies of the system. Results presented are valuable in the design of this type of gyroscope as the exact resonant conditions and the sensitivity of the system parameters play important roles in the dynamic performance of gyroscopes.

1. Introduction

Due to the wide range of the applications of the vibrating mass gyroscopes; they are being used in many navigational applications, namely, aerospace, marine and automobile industries. Hence, the detailed study of such systems has always been of great interest to engineers and researchers.

In most of these types of gyroscopes, the bending and torsional vibrations are coupled. The theory of coupled flexural–torsional vibrations for thin-walled beams was first developed by Timoshenko and Young [1]. The free flexural/torsional vibration of an Euler–Bernoulli beam with a rigid tip mass was studied by Oguamana [2]. He presented explicit expressions for the frequency equation, mode shapes and their orthogonality relationship and investigated the effects of different parameters on the fundamental frequencies of the system. Salarieh and Gorashi [3] continued his work, but used the Timoshenko beam theory. They studied the effects of the shear deformation and the rotary inertia on the free vibration response of a Timoshenko beam with a rigid tip mass. Gokdag and Kopmaz [4] extended the work of

---

[*] Corresponding author. Tel.: 1 905 721 8668x2600; fax: 1 905 721 3370.
E-mail address: ezadeh@uoit.ca (E. Esmailzadeh).

0022-460X/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsv.2009.06.021

Nomenclature

b	width of the beams
b_M	rocking-mass width
C_{ij} , C'_{ij}	elements of matrices
d_{31}	piezoelectric constant of actuators
E	elastic modulus of beams
E_p	elastic modulus of actuators
EI_b	flexural rigidity of the beams
EI_p	flexural rigidity of the actuator
G	beams shear modulus
GJ_b	torsional rigidity of the beams
GJ_p	torsional rigidity of the actuator
h_M	Rocking-mass height
i	$\sqrt{-1}$
I_{xb}	mass moment of inertia of the beams
I_{xp}	mass moment of inertia of the piezoelectric actuator
l	length of rocking-mass
l_1	start position of piezoelectric actuator
l_2	end position of piezoelectric actuator
L_i	length of ith beam
M	rocking-mass
M_P	piezoelectric actuator control moment
$Q_n(x)$	amplitude of torsion
$P_n(x)$	amplitude of bending
s_1	position of point P on beam 1
T	total kinetic energy of the system
t_b	beams thickness
t_p	piezoelectric layers thickness
T_{1b}	translational kinetic energy of the four beams due to the translational motion
T_{1M}	translational kinetic energy of the rocking-mass due to the translational motion
T_{2b}	translational kinetic energy of the four beams due to the rotational motion
T_{2M}	translational kinetic energy of the rocking-mass due to the rotational motion
V	total potential energy of the system
$v(t)$	voltage applied to actuators
w_i	bending of ith beam
W_{nc}	total non-conservative work done on the system
θ_i	torsion of ith beam
ρ_b	mass per unit length of the beams
ρ_p	mass per unit length of the piezoelectric actuator
ψ_{i}, θ_i	angle of rotations of ith beam
Ω_1	rotation about X_i-axis
Ω_2	rotation about Y_i-axis
Ω_3	rotation about Z_i-axis
ω	vibration frequency
ω_{ix}	angular velocity component of ith beam in x-direction
ω_{iy}	angular velocity component of ith beam in y-direction
ω_{iz}	angular velocity component of ith beam in z-direction

Oguamana [2] by studying the coupled flexural/torsional vibrations of a beam with either the tip or in the span mass attachments.

In a series of studies, Jalili and his research team [5–7] worked on the vibrating gyroscopic systems experiencing coupled flexural/torsional vibrations. Their first work was to develop a thorough modeling framework for vibrating gyroscopes subjected to general support motion by considering both the flexural and torsional vibrations [5]. In a subsequent work, they include a novel piezoelectric actuation for the vibrating beam gyroscope, which was modeled as an Euler-Bernoulli beam with a tip load subjected to the base rotation. They investigated the effect of the cross-axis in single beam vibratory gyroscopes [6], and also the influence of the substrate motion on the performance of the ring microgyroscopes [7].

Although vibrating beam gyroscopes are becoming the most widely used gyroscopes in many applications [6], but they possess a very important drawback, which produces the cross-coupling error in the measurements [6,8]. The vibrating beam gyroscope is typically used to measure the rotational rate around one of the axes. In practice, however, there are always some secondary rotations present in the system. These secondary base rotations can produce significant errors in the measurement of the gyroscope output (cross-axis error). The gyroscopic output increases significantly even for a small secondary rotation. This increased output could be interpreted as the gyroscope output due to the primary base rotation and can hence, develop errors in the measurement [6].

In spite of single beam gyroscopes, the rocking-mass gyroscope does not have those drawbacks, and can accurately measure the rate of rotation. Due to the complexities involved in the modelling and performance analysis of this kind of gyroscope, only few studies have been carried out in this area. The fabrication and design of a rocking-mass gyroscope was studied by Tang and Gutierrez [9], but the operating principle of the device was not fully discussed. Royle and Fox [10] presented an analysis of the mechanics of an oscillatory rate gyroscope that is actuated and sensed using thin piezoelectric actuators and sensors. A modeling framework for these systems, which forms the basis of this paper, is an extension to the work reported by Bhadbhade [11].

The present research undertakes the vibration analysis of a rocking-mass gyroscope, which comprises of a rotating rigid substrate and an assembly of four cantilever beams with a rigid mass attached to them in the middle, as shown in Fig. 1. The objective of the research is to develop a detailed mathematical modeling of the system. The governing equations of motion, using the extended Hamilton's principle, are derived. Since the closed-form solutions can serve as the benchmarks for validating the results obtained from either the numerical calculations or experimental results, the closed-form equations are developed for the frequency characteristic equations of the system for either a fixed supporting base or a rotating one. These exact equations are very important and useful, since their solutions would not only provide exact