Nonlinear Analysis 71 (2009) e2814–e2823

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

An approximate Malliavin weight for Variance Gamma process: Sensitivity analysis of European style options

Derviş Bayazıt *, Craig A. Nolder

Florida State University, Department of Mathematics, 208 Love Building 1017 Academic Way Tallahassee, FL 32306-4510, United States

A R T I C L E I N F O

MSC:
60H07
65C05
11K45
60J75
65T50

Keywords:
Malliavin calculus
Monte Carlo simulations
European options
Acceptance–rejection method
Compound Poisson approximation
Sensitivity analysis
Fast Fourier transform
Variance Gamma process

A B S T R A C T

The main objective of this work is to find an approximate Malliavin weight in order to calculate the delta of European style options where the underlying asset is modelled by a Variance Gamma process. We use a Malliavin integration by parts formula for a compound Poisson process. In order to apply this formula, a compound Poisson approximation of the Variance Gamma process is used. We calculate deltas using Monte Carlo simulations. An acceptance–rejection algorithm is used to generate random numbers for the approximated VG process.

1. Introduction

In the past years, following the pioneering papers [1,2], Malliavin calculus (stochastic variational calculus) has been used extensively in the field of mathematical finance. Main topics discussed in terms of application of Malliavin calculus mostly include the computation of sensitivities and of conditional expectations. Earlier works of these applications focused on the lognormal type diffusion processes. Lately, jump type market models have been used by practitioners and researchers. Application of the Malliavin calculus in markets driven by Lévy processes, specifically by pure jump or jump diffusion processes are discussed in [3,4] and [5,6]. In [6], the Malliavin integration by parts formula is derived under the assumption that the law of random variables is absolutely continuous with respect to the Lebesque measure and smooth enough so that the first derivative exists and is continuous.

The main purpose of this paper is to derive an integration by parts formula when the stock prices follow an exponential Variance Gamma (VG) process [7] in terms of the method introduced in [6]. However, increments of the VG process do not satisfy the necessary assumption of smoothness of the density. In order to overcome this problem, we approximate the VG process by using the compound Poisson approximation, [8]. Thus, we are able to derive an explicit formula for the deltas of European call and digital options. We also used fast Fourier transform (FFT) method in order to compute delta by the method appearing in [9]. We measured the performance of our results in Malliavin approach in terms of the FFT computations of the deltas. For more complex financial derivatives like Asian options we do not have the characteristic function information. Therefore, in this case the implementation of the inverse Fourier method is not possible and a method such as Malliavin approach is necessary.

* Corresponding author. Tel.: +1 850 591 4157; fax: +1 850 644 4053.
E-mail addresses: dbayazit@math.fsu.edu (D. Bayazıt), nolder@math.fsu.edu (C.A. Nolder).

0362-546X/$ – see front matter
doi:10.1016/j.na.2009.06.063

The paper is organized as follows: In Section 1 we give the analytic formulas that we obtain for the deltas in terms of the characteristic function of the log stock price. We show the explicit calculations in order to derive the pricing function for the digital option by using the inverse Fourier transform method. In the following section, approximation of the VG process and properties of the resulting compound Poisson process are discussed. In the subsection, an acceptance–rejection algorithm is introduced for the specific density of the jump size distribution. In Section 3, preliminaries of Malliavin calculus for simple functionals and the integration by parts formula of [6] are given. In Section 4, numerical schemes, formulas and the graphical results are presented. We use a finite difference method on the exact[1] simulation of the VG process in order to measure the performance of the FFT method which we use as our benchmark. The explicit formula for delta and its derivation by using the Malliavin approach are presented in this section, as well. Results show that as the approximation parameter ε gets smaller, the approximation of delta gets better. However, this leads to a slower convergence of the Monte Carlo simulations.

2. FFT for delta of digital and European call options

In this section we will give a closed form solution for the delta of a digital option. In [9], the price of a European call option is obtained by inverting the characteristic function of the logarithm of the stock price S_t. If $Y_T = \log(S_T)$ is the log stock price at time T with the risk neutral density $\rho_T(y)$, then the characteristic function of Y_T is defined as

$$\phi_{Y_T}(u) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}us} \rho_T(s)\mathrm{d}s.$$

The price of a European call option is given by the following integral:

$$C_T(k) := \int_k^{\infty} \mathrm{e}^{-rT}(\mathrm{e}^y - \mathrm{e}^k)\rho_T(y)\mathrm{d}y,$$

where k is the log of the strike price. Since $C_T(k)$ approaches S_0 as k tends to $-\infty$, the call pricing function is not square integrable. In [9], a modified call price

$$C_T^m(k) = \mathrm{e}^{\alpha k} C_T(k)$$

is introduced, where $\alpha > 0$ is called as the damping coefficient. However, $C_T^m(k)$ is a square integrable function in $k \in (-\infty, \infty)$. The Fourier transform of $C_T^m(k)$ is: $\psi_T(\nu) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\nu k} C_T^m(k)\mathrm{d}k$. Therefore, an integral expression for the call option price is obtained by the inverse Fourier transform of $\psi_T(\nu)$,

$$C_T(k) = \frac{\mathrm{e}^{-\alpha k}}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\nu k}\psi_T(\nu)\mathrm{d}\nu = \frac{\mathrm{e}^{-\alpha k}}{\pi} \int_0^{\infty} \mathrm{e}^{-\mathrm{i}\nu k}\psi_T(\nu)\mathrm{d}\nu. \tag{2.1}$$

Thus, in order to evaluate (2.1) numerically one needs to know $\psi_T(\nu)$ and it is determined as follows,

$$\begin{aligned}\psi_T(\nu) &= \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\nu k} \int_k^{\infty} \mathrm{e}^{\alpha k}\mathrm{e}^{-rT}(\mathrm{e}^y - \mathrm{e}^k)\rho_T(y)\mathrm{d}y\mathrm{d}k \\ &= \frac{\mathrm{e}^{-rT}\phi_{Y_T}(\nu - (\alpha+1)\mathrm{i})}{\alpha^2 + \alpha - \nu^2 + \mathrm{i}(2\alpha+1)\nu}. \end{aligned} \tag{2.2}$$

In the digital option case, the pricing function and its modification with α are

$$D_T(k) := \int_k^{\infty} \mathrm{e}^{-rT}\rho_T(y)\mathrm{d}y, \qquad D_T^m(k) = \mathrm{e}^{\alpha k} D_T(k),$$

respectively. Thus,

$$D_T(k) = \frac{\mathrm{e}^{-\alpha k}}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\nu k}\xi_T(\nu)\mathrm{d}\nu = \frac{\mathrm{e}^{-\alpha k}}{\pi} \int_0^{\infty} \mathrm{e}^{-\mathrm{i}\nu k}\xi_T(\nu)\mathrm{d}\nu. \tag{2.3}$$

Therefore, in order to evaluate (2.3) numerically one needs to know $\xi_T(\nu)$ and it is determined as follows:

$$\begin{aligned}\xi_T(\nu) &= \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\nu k} \int_k^{\infty} \mathrm{e}^{\alpha k}\mathrm{e}^{-rT}\rho_T(y)\mathrm{d}y\mathrm{d}k \\ &= \int_{-\infty}^{\infty} \mathrm{e}^{-rT}\rho_T(y) \int_{-\infty}^{y} \mathrm{e}^{\alpha k}\mathrm{e}^{\mathrm{i}\nu k}\mathrm{d}k\mathrm{d}y \\ &= \int_{-\infty}^{\infty} \mathrm{e}^{-rT}\rho_T(y) \left.\frac{\mathrm{e}^{(\alpha+\mathrm{i}\nu)k}}{\alpha+\mathrm{i}\nu}\right|_{-\infty}^{y} \mathrm{d}y \end{aligned}$$

[1] i.e., without any approximation. See Section 5.1 for details.