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Sensitivity analysis for estimation of inertial parameters of multibody mechanical systems

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ABSTRACT

As an important issue, we first address in this paper the problem of a regression matrix with non-homogeneous physical units in the parameter estimation of multibody systems. This matrix contains the most important information about the motion of the system. Attention must be paid to this before implementing any parameter estimation algorithm due to the unit inconsistency in matrix multiplication required in such algorithms. A procedure to treat this problem in a proper way will be proposed. An experiment on a six degrees of freedom (DOF) robotic device, whose reference link follows a desired trajectory, is performed. The data collected from the experiment are then used for sensitivity analysis of inertial parameters based on the unit-homogenized regression matrix of the system. In this way, we characterize the influence of each selected inertial parameter on the dynamics of the system using unit-consistent mathematical manipulations.

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1. Introduction

Parameter estimation is of great importance in analysis and design of multibody systems. In this context, various numerical approaches have been used to identify inertial parameters based on measurement data collected from experiments, e.g. [8,15,16]. The majority of contributions devoted to this subject have focused more on some specific robotic applications, e.g. [2,6,7,9,10,18,19]. Compared to the area of robotics, relatively little attention has been paid to the parameter estimation in the general area of multibody systems [3,14,17]. However, this is an area of utmost importance since accurate knowledge of the inertial parameters is required for realistic dynamic simulations and analyses.

The modeling of multibody systems usually involves the development of dynamic equations. The structure of the model and the equations of motion can usually be determined but the inertial parameters are not necessarily known in advance. Standard inertial parameters associated with body i consist of mass m_i, first moments of inertia $\mathbf{c}_i = [c_{x_i}, c_{y_i}, c_{z_i}]^T$, and moments and products of inertia $\mathbf{J}_{V_i} = [J_{xx_i}, J_{yy_i}, J_{zz_i}, J_{xy_i}, J_{xz_i}, J_{yz_i}]^T$ with units of kg, kg m and kg m^2, respectively. The elements of vectors $\mathbf{c}_i$ and $\mathbf{J}_{V_i}$ are defined with respect to the reference frame of body i with coordinates x, y and z. With proper selection of the location of the origin of this body reference frame, the dynamic equations will be linear in terms of these inertial parameters. For parameter estimation algorithms, the system of equations of motion is first generated and then, usually reformulated in terms of the inertial parameters. Consider a system of n bodies with total f degrees of freedom (DOF) and r inertial parameters ($r = 10n$ for a general system). The minimal form of the dynamic equations can be

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formulated and written as, [8,17],

$$\bar{\mathbf{Y}}\mathbf{x} = \bar{\mathbf{Q}}, \tag{1}$$

where $\bar{\mathbf{Y}}$ is an $f \times r$ matrix which contains the coefficient expressions associated with the inertial parameters in the dynamic equations. The components of $\bar{\mathbf{Y}}$ are functions of the position, velocity, and acceleration of the bodies. Vector $\mathbf{x}$ with dimension $r \times 1$ is the vector of *actual inertial parameters* which basically collects the standard inertial parameters of the bodies of the system, and $\bar{\mathbf{Q}}$ is the vector of applied forces and/or torques. Then, the system is moved to follow a certain trajectory and the position, velocity, and acceleration data are measured at some points. The set of identification equations for h measurement points can be written as

$$\mathbf{A}\mathbf{x} = \boldsymbol{\tau} \quad \text{with } \mathbf{A} = [\bar{\mathbf{Y}}_1^T \ \ldots \ \bar{\mathbf{Y}}_h^T]^T, \ \ \boldsymbol{\tau} = [\bar{\mathbf{Q}}_1^T, \ldots, \bar{\mathbf{Q}}_h^T]^T, \tag{2}$$

where $\mathbf{A}$ is the *regression matrix* of dimension $hf \times r$, whose elements are all known quantities, and $\boldsymbol{\tau}$ is the $hf \times 1$ vector of applied forces and/or torques determined at the measurement points. The number of measurement points should be large enough to ensure a reliable parameter estimation procedure [9,10].

Parameter estimation algorithms are basically based on mathematical manipulations of arrays. Such operations are required in many numerical methods (e.g. singular value decomposition (SVD), QR). If the elements of $\mathbf{A}$ have different physical units, then the evaluation of such matrix products would involve the addition of terms with different units. This, however, physically makes no sense. Consequently, the identified inertial parameters, as the result of these approaches, may not be reliable and meaningful. In this paper, we propose our procedure to homogenize the units of the regression matrix of a system in order to avoid the addition of terms with different units. Then, a sensitivity analysis of a dual-pantograph device is performed in terms of some inertial parameters. We assess the effect of various inertial parameters on the dynamics based on the unit-homogenized regression matrix of the system. This can be of great importance in the analysis and design of mechanical systems. Because of the kinematic topology of a mechanical system, the inertial parameters influence the dynamics of the system with different levels of contribution [4,14]. Based on a sensitivity analysis, we investigate how the dynamics of a system is influenced with respect to the variation of inertial parameters. Those parameters which show no contribution may be neglected during the analysis and design step. This helps the analyst to pay more attention to the inertial parameters that have greater effect.

2. Unit homogenization of matrix A

For the better illustration of the approach, we assume that each independent coordinate is of the rotational type and therefore, vector $\boldsymbol{\tau}$ in equation $\mathbf{A}\mathbf{x} = \boldsymbol{\tau}$ would be an $hf \times 1$ vector with units of (N m). The procedure which will be explained here can be used in the same way when each independent coordinate is of the translational type, but vector $\boldsymbol{\tau}$ would contain applied forces with units of (N). However, for multibody systems represented with independent coordinates with mixed units, the proposed approach may be extended to include independent coordinates of both rotational and translational types.

As the first step of our approach, we rearrange the columns of matrix $\mathbf{A}$ in such a way that the columns which correspond to the same type of the inertial parameters would be grouped together. Since we deal with three groups of inertial parameters, one can establish three submatrices in $\mathbf{A}$. We first take the columns of $\mathbf{A}$ corresponding to the mass of each body and put them in a matrix $\mathbf{C}_m$ of dimension $hf \times n$. Then, following the same strategy for the other two groups of the inertial parameters, $\mathbf{c}$ and $\mathbf{J}_V$, we build an $hf \times 3n$ matrix $\mathbf{C}_c$ and an $hf \times 6n$ matrix $\mathbf{C}_J$ associated with $\mathbf{c}$ and $\mathbf{J}_V$, respectively. These three submatrices can now be used to build another form of the regression matrix as

$$\mathbf{C} = [\mathbf{C}_m \ \ \mathbf{C}_c \ \ \mathbf{C}_J]. \tag{3}$$

In a similar manner, the vector of the inertial parameters $\mathbf{x}$ is partitioned to three subvectors $\mathbf{z}_m, \mathbf{z}_c$, and $\mathbf{z}_J$, and rearranged in a new vector $\mathbf{z}$ as

$$\mathbf{z}_{(r\times 1)} = [\mathbf{z}^T_{m(n\times 1)}, \mathbf{z}^T_{c(3n\times 1)}, \mathbf{z}^T_{J(6n\times 1)}]^T, \tag{4}$$

where

$$\mathbf{z}_m = [m_1, \ldots, m_n]^T, \quad \mathbf{z}_c = [\mathbf{c}_1^T, \ldots, \mathbf{c}_n^T]^T, \quad \mathbf{z}_J = [\mathbf{J}_{V_1}^T, \ldots, \mathbf{J}_{V_n}^T]^T. \tag{5}$$

With the new rearranged regression matrix $\mathbf{C}$ and the vector of inertial parameter $\mathbf{z}$, the new set of equations can be written from Eq. (2) as

$$\mathbf{A}\mathbf{x} = \boldsymbol{\tau} \Rightarrow \mathbf{C}\mathbf{z} = \boldsymbol{\tau} \Rightarrow \mathbf{C}_m\mathbf{z}_m + \mathbf{C}_c\mathbf{z}_c + \mathbf{C}_J\mathbf{z}_J = \boldsymbol{\tau} = \boldsymbol{\tau}_m + \boldsymbol{\tau}_c + \boldsymbol{\tau}_J, \tag{6}$$

where the right-hand side vectors $\boldsymbol{\tau}_m$, $\boldsymbol{\tau}_c$, and $\boldsymbol{\tau}_J$ are not known but their summation is equal to $\boldsymbol{\tau}$. Since the elements of each of $\mathbf{z}_m$, $\mathbf{z}_c$, and $\mathbf{z}_J$ have the same unit, their corresponding coefficient matrices $\mathbf{C}_m$, $\mathbf{C}_c$, and $\mathbf{C}_J$ contain terms with homogeneous units. We can now use the singular value decomposition to decompose these matrices as

$$\mathbf{C}_m = \mathbf{U}_m\mathbf{W}_m\mathbf{V}_m^T, \quad \mathbf{C}_c = \mathbf{U}_c\mathbf{W}_c\mathbf{V}_c^T, \quad \mathbf{C}_J = \mathbf{U}_J\mathbf{W}_J\mathbf{V}_J^T. \tag{7}$$

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