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On the sensitivity analysis of camera calibration from images of spheres

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ABSTRACT

This paper presents a novel sensitivity analysis of camera calibration from images of spheres. We improve the accuracy of a conic matrix and hence the accuracy of calibration results by eliminating the ambiguity of the conic orientation that arises from the nature of a circular-ellipse. In addition, relationships between the difference in length of the long and short axes of a conic and other parameters of the conic and the camera are investigated and demonstrated through parametric study and experimental analysis. By utilizing these relationships, we establish novel guidelines that can be followed to obtain better calibration results.

1. Introduction

The purpose of camera calibration is to recover intrinsic and extrinsic parameters, which is a preliminary step for further developing a visual tracking system or a visual servoing system [13,23]. Using calibration objects is a common approach to camera calibration [18]. Based on the geometry of the camera model and the object, intrinsic and extrinsic parameters can be recovered. Spheres have been considered as calibration objects, because they have consistent appearance when observed from an arbitrary direction. Moreover, compared with other types of calibration objects, spheres are especially efficient for the calibration of multi-camera visual systems. For example, special patterns with known metric structures, such as grids, might not be simultaneously visible in the view of each camera. For mobile robots with vision systems, spheres can also be used as markers as an alternative for the robots to localize themselves.

Some possible approaches to camera calibration from images of spheres have been proposed in previous literatures. Daucher et al. [2,4] have proposed to compute the aspect ratio, the optical center, and the focal length of a camera based on the fact that the major axis of the sphere projection passes through the optical center of the image. They use four parameters to model the projection matrix of a camera by assuming that the skew coefficient is zero. Teramoto et al. [9,11,14–16,19,21,22] have proposed to recover camera parameters based on the relations between conics of spherical objects and the image of the absolute conic (IAC). Teramoto et al. [9,11] use non-linear approaches in order to solve for camera parameters by minimizing geometric and algebraic errors. Teramoto and Xu [9] have suggested a need for accurate initial estimation of the intrinsic parameters in order to minimize those errors, while [11] does not make any assumption on the parameters to be estimated.

Ying et al. [14–16,19,21,22] have used linear approaches, named as scalar and orthogonal approaches, respectively, to solve for camera parameters. The scalar approach makes use of the algebraic constraint on the IAC arising from an image conic. The orthogonal approach interprets the algebraic constraint geometrically. It is observed that a conic is tangent to the IAC at two points, which are on the line coincident with the vector that is dependent on the rotation between the camera and the world frames.

Regardless of the approach used, the accuracy of camera calibration from images of spheres depends largely on the accuracy of conic matrices. Entries of conic matrices indicate the influence of camera parameters on the projection of spheres. Little work has been done on the sensitivity analysis of the calibration method based on the conic extraction. In general, errors that can be presented in conic matrices are unavoidable, and they can hence result in unacceptable recovered camera parameters.

In this paper, we improve the accuracy of a conic matrix by eliminating the ambiguity of the conic orientation that arises from the nature of a circular-ellipse. An assumption is made on the orientation using the fact that the major axis of a conic passes through the optical center of the image. Such an assumption, as shown in the paper, does result in the orientation to be uniquely and correctly determined. In addition, we investigate relationships between the difference in length of the long and short axes of a conic and other parameters of the conic and the camera. Through these relationships, we can hence avoid undesirable conditions for camera calibration or to reduce their impacts on calibration results.

This paper is organized as follows. Section 2 provides an overview of camera calibration from images of spheres. In Section 3,

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we present the determination of conic parameters and the computation of the entries of conic matrices. In Section 4, relationships between the difference in length of the long and short axes of a conic and other parameters of the conic and the camera are investigated and demonstrated. Section 5 presents experimental results, which are in support of those relationships observed in Section 4. Section 6 presents the concluding remarks.

2. Camera calibration from images of spheres

2.1. Pinhole camera model

Under the pinhole camera model [1,8], a point $\mathbf{X} = (x, y, z, 1)$ in the world frame is mapped to the point $\mathbf{x} = (u, v, 1)$ on the image plane by

$$\mathbf{x} = P\mathbf{X}, \tag{1}$$

where

$$P = KR[I|t]. \tag{2}$$

In Eq. (2), I is a 3×3 identical matrix, and K is the matrix of intrinsic parameters that define the image geometry of the camera.

$$K = \begin{bmatrix} f_u & s & u_o \\ 0 & f_v & v_o \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

In the matrix K, $f_u = fm_u$ and $f_v = fm_v$ represent the focal length f (m) in terms of pixel dimension in u and v directions, where m_u and m_v are numbers of pixels per meter in each direction. (u_o, v_o) is the optical center of the image. $s = \sin(\pi/2 - \beta)f_u$ is the skew coefficient, where β is the angle between the u and v sensor axes. s equals to 0 for $\beta = \pi/2$, which means that pixels are rectangles.

R and t in Eq. (2) are extrinsic parameters that relate the camera frame to the world frame. R ($RR^T = I$) is a 3×3 rotational matrix representing the orientation of the camera frame (X_c–Y_c–Z_c) with respect to the world frame (X–Y–Z). t is the translation of the camera frame with respect to the world frame.

2.2. Spheres in the pinhole camera model

Fig. 1 shows the pinhole geometry of a single camera view with a sphere projected onto the image plane π. The cone with vertex $\mathbf{O}$ positioned at the origin of the camera frame (X_c–Y_c–Z_c) is the smallest cone that contains the sphere. The surface of the cone is a quadric $\mathbf{Q}$. By setting the origin of the world frame (X–Y–Z) at $\mathbf{O}$ and by aligning the Z-axis of the world frame with the revolution axis of the cone, a point $\mathbf{X}$ on the surface $\mathbf{Q}$ with the world coordinates of $\mathbf{X} = (x, y, z, 1)^T$ satisfies the equation given as

$$(x^2 + y^2)\cos^2\frac{\alpha}{2} - z^2\sin^2\frac{\alpha}{2} = 0, \tag{4}$$

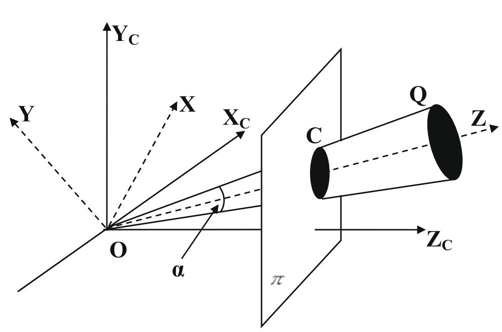

Fig. 1. A sphere is projected onto the image plane π as an image conic $\mathbf{C}$, according to the pinhole camera geometry.

where α is the aperture of the cone. Eq. (4) can be equivalently represented in the matrix form as

$$\hat{\mathbf{X}}^T\hat{\mathbf{Q}}\hat{\mathbf{X}} = 0, \tag{5}$$

where $\hat{\mathbf{X}}$ is the inhomogeneous form of $\mathbf{X}$, and

$$\hat{\mathbf{Q}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\lambda^2 \end{bmatrix}, \tag{6}$$

with $\lambda = \tan(\alpha/2)$.

The intersection of the quadric $\mathbf{Q}$ with the image plane π is an image conic $\mathbf{C}$, which is defined as

$$\mathbf{x}^T\mathbf{C}\mathbf{x} = 0, \tag{7}$$

$\mathbf{C}$, the conic matrix, is a 3×3 symmetric matrix with the entry $\mathbf{C}_{(3,3)} = 1$, and

$$\mathbf{C} = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & 1 \end{bmatrix}. \tag{8}$$

If $\hat{\mathbf{X}}$ satisfies Eq. (5), $\mathbf{x}$, the mapped point of $\hat{\mathbf{X}}$ on the image plane, satisfies Eq. (7).

In Fig. 1, the camera frame (X_c–Y_c–Z_c) shares the same origin with the world frame (X–Y–Z), which results in $t = (0, 0, 0)^T$. Therefore, from Eq. (1), we can write

$$\mathbf{x} = P\mathbf{X} = KR[I|\mathbf{0}]\mathbf{X} = KR\hat{\mathbf{X}}. \tag{9}$$

By re-arranging the above equation, we have

$$\hat{\mathbf{X}} = R^{-1}K^{-1}\mathbf{x}. \tag{10}$$

By substituting Eq. (10) into Eq. (5), we have the following:

$$\mathbf{x}^TK^{-T}R^{-T}\hat{\mathbf{Q}}R^{-1}K^{-1}\mathbf{x} = 0. \tag{11}$$

The expression of $\mathbf{C}$ can be finally obtained from Eqs. (7) and (11), which is

$$\kappa\mathbf{C} = K^{-T}R^{-T}\hat{\mathbf{Q}}R^{-1}K^{-1}, \tag{12}$$

where κ is a non-zero scalar.

2.3. Recovery of camera parameters

Intrinsic parameters are recovered based on the relations between conics of spherical objects and the IAC. The IAC is a point conic on the image plane at infinity that is invariant to rotation and translation. ω, the matrix for the IAC, is a 3×3 symmetric matrix given as

$$\omega = K^{-T}K^{-1} = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{12} & \omega_{22} & \omega_{23} \\ \omega_{13} & \omega_{23} & \omega_{33} \end{bmatrix}. \tag{13}$$

From Eqs. (6), (12) and (13), we can obtain

$$\kappa\mathbf{C} = \omega - vv^T, \tag{14}$$

where

$$v = \sqrt{1 + \lambda^2}K^{-T}r_3, \tag{15}$$

and r_3 is the third column of the rotation matrix R. Eq. (14) defines the relation between an image conic $\mathbf{C}$ and the IAC.

We need at least three spheres in order to recover the intrinsic parameters of one camera. This can be seen from the numbers of unknowns on both sides of Eq. (14). The number of unknowns in $\mathbf{C}$ is 5 as shown in Eq. (8). Together with the scalar κ, the number of unknowns for N conics is $6N$. From Eq. (13), we have 6