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The combination of self-organizing feature maps and support vector regression for solving the inverse ECG problem

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ABSTRACT

Noninvasive electrical imaging of the heart aims to quantitatively reconstruct transmembrane potentials (TMPs) from body surface potentials (BSPs), which is a typical inverse problem. Classically, electrocardiography (ECG) inverse problem is solved by regularization techniques. In this study, it is treated as a regression problem with multi-inputs (BSPs) and multi-outputs (TMPs). Then the resultant regression problem is solved by a hybrid method, which combines the support vector regression (SVR) method with self-organizing feature map (SOFM) techniques. The hybrid SOFM–SVR method conducts a two-step process: SOFM algorithm is used to cluster the training samples and the individual SVR method is employed to construct the regression model. For each testing sample, the cluster operation can effectively improve the efficiency of the regression algorithm, and also helps the setup of the corresponding SVR model for the TMPs reconstruction. The performance of the developed SOFM–SVR model is tested using our previously developed realistic heart-torso model. The experiment results show that, compared with traditional single SVR method in solving the inverse ECG problem, the proposed method can reduce the cost of training time and improve the reconstruction accuracy in solving the inverse ECG problem.

1. Introduction

In one typical form of inverse electrocardiography (ECG) problem [1–4], the heart's electrical activity is noninvasively imaged from the analysis of the body surface potentials (BSPs). The inverse ECG problem is usually solved using model-based schemes, including activation-based models [5–7] or potential-based models [8–12] (such as epicardial, endocardial, or trans-membrane potentials). Practically, due to the ill-posed property, the inverse ECG problem will be solved in an indirect manner and the term "regularization" is usually used to relax the ill-posedness of the problem [13–15]. Because of suffering from system noise, such as geometry noise and measurement noise, these regularization methods still cannot always guarantee the quality of the inverse solution. In this work, we attempt to study the inverse ECG problem using a non-regularization based method. Here, an alternative method, Support Vector Regression (SVR) method [16–18], was proposed to solve the inverse ECG problem. In this scheme, the inverse ECG problem is treated as a regression problem with multi-inputs (BSPs) and multi-outputs (TMPs) during the solution procedure, and the SVR technique will be used to solve the nonlinear regression problem [19,20].

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Recently, the SVR model has been successfully applied to solving prediction problems outside of the ECG field [21,22]. This research will explore its new application in ECG inverse problem. As in other prediction areas, the prediction accuracy remains a prime issue of concern; for the prediction/diagnosis of cardiac diseases, the prediction accuracy of the model is particularly critical in term of patient's health. Feature extraction plays an important role in improving prediction accuracy in the process of developing an effective SVR model. In our previous studies [19], the kernel principal analysis (KPCA) is proposed to preprocess the original input for feature extraction, referred to as KPCA–SVR, which perform better than that without feature extraction (traditional single SVR) in terms of the reconstruction of TMPs. Compared with single SVR model, the hybrid models integrated with multiple methods usually show better performance in prediction. For example, the combination of maximum margin clustering (MMC) method and SVR was proposed to solve the inverse ECG Problem [20], which can enhance the computation efficiency of training the SVR model. In addition, the self-organizing feature map (SOFM) is an unsupervised and competitive learning algorithm, which is another promising clustering technique [23]. Combining the SOFM with SVR or LS-SVM, the hybrid method has the potential to find better inverse solutions than using a single SVR model [24–26], which can improve the prediction accuracy of the traditional SVR method and to reduce its long training time. Cao [27] also combined the SOFM with SVM to deal with the time series forecasting problem. Huang and Tsai [28] hybridized SVR with the self-organizing feature map (SOFM) technique and a filter-based feature selection to reduce the cost of training time and improve prediction accuracies. And Li [29] combined the SOFM with SVM for face recognition, which lessened the classifier cost of SOM-SVM to one-tenth of that with SVM. Compared with MCC algorithm, SOFM can provide more reliable clustering results, and thus has been applied in a wider range of domains. In all, it was reported that a hybrid SVR method can achieve a higher performance level than the traditional single SVR.

In this study, we attempt to investigate the SOFM–SVR method for reconstructing the cardiac TMPs from BSPs. To train a SOFM–SVR model, a two-step process is devised and its performance is compared with those of traditional single SVR method. A short description of this work was reported in a recent conference [30]. This submission has undergone substantial extension and offers more experiment results.

2. Theory and methodology

2.1. Support vector regression method

A brief description of the support vector regression (SVR) algorithm is given here, and details can be found in the reference paper [16].

Given a training set (x_i, y_i), $i = 1, 2, m$, where the input variable $x_i \in R^n$ is the n-dimensional vector; and the response variable $y_i \in R$ is the continuous value. SVR builds the linear regression function as the following form:

$$f(X, W) = W^T X + b. \tag{1}$$

Based on Vapnik's linear ε-Insensitivity loss (error) function, the linear regression $f(X, W)$ is estimated by simultaneously minimizing $\|W\|^2$ and the sum of the linear ε-Insensitivity losses. The parameter C is determined by the user, which balances a trade-off between an approximation error and the weights vector norm $\|W\|$.

$$|y - f(X, W)|_\varepsilon = \begin{cases} 0, & \text{if } |y - f(X, W)| \le \varepsilon \\ |y - f(X, W)| - \varepsilon, & \text{otherwise} \end{cases} \tag{2}$$

$$R_{W,\xi,\xi^*} = \frac{1}{2}\|W\|^2 + C\sum_{i=1}^{m}(\xi + \xi^*) \tag{3}$$

under constraints:

$$\begin{cases} \left(W^T X_i + b\right) - y_i \le \varepsilon + \xi_i \\ y_i - \left(W^T X_i + b\right) \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0, \quad i = 1, 2, \ldots, m \end{cases} \tag{4}$$

where ξ_i and ξ_i^* are slack variables, one for exceeding the target value by more than ε, and the other for being more than ε below the target. As with procedures applied to SVM classifiers [17,18], this constrained optimization problem is handled by applying the Lagrange method, and the Karush–Kuhn–Tucker condition. Through optimization, the optimal desired weights vector of the regression function can be obtained. The constrained optimization problem shown in Eq. (3) can be further restated as the following equation

$$\begin{gathered} f(x, \alpha_i, \alpha_i^*) = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)K(x_i, x) + b \\ \text{subject to } \sum_{i=1}^{n}(\alpha_i - \alpha_i^*) = 0 \quad \text{and} \quad 0 \le \alpha_i, \alpha_i^* \le C \end{gathered} \tag{5}$$