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Sensitivity analysis of Matching Pennies game

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ABSTRACT

In this paper, we have discussed the results of sensitivity analysis in a payoff matrix of the Matching Pennies game. After representing the game as a LP model, the sensitivity analysis of the elements of the payoff matrix is presented. The game value and the optimal strategies for different values of parameters are determined and compared.

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1. Introduction

In many strategic situations, one person prefers a "match", whereas the other prefers a "mismatch". As an example, a soccer goalie prefers to dive in the same direction as the penalty kick goes, whereas the kicker has opposite preferences. As a result, there is no Nash equilibrium in pure strategies. Similarly, while a business manager hopes that an audit occurs only in cases when preparations have been made, the auditor prefers that audits catch sloppy record keepers. These games can be approximated as zero-sum Matching Pennies games (MP)[1] [1]. Indeed, the Matching Pennies game, a simplified version of the more popular Rock, Paper, Scissors, schematically represents competitions between organisms which endeavor to predict each other's behavior [2].

As in Rock, Paper, Scissors, in order for a player to play optimally in iterated MP competitions, he/she should produce unpredictable sequences of choices and attempt to detect nonrandomness in the opponent's choices. This is the Nash equilibrium of MP: "Against a choice-randomizing opponent, the best strategy is to randomize one's own choices" [2].

The Matching Pennies game has been studied and analyzed for decades. For instance, in the literature on evolutionary games, the instability of the equilibrium point (which is a mixed strategy equilibrium) in MP is well known[2] [3]. Also this game has been analyzed in the realm of behavioral game theory.[3] It has been shown that humans [4] and other primates [5] can learn to compete efficiently in MP [2].

Although suboptimal sensitivity to one's own payoffs in mixed strategy games has been observed in human players [6,7], the Nash theorem [8–10] guarantees that there exists a unique optimal solution (or equivalently a unique equilibrium point) for any n-person game, including MP.

In this paper, by introducing an analytical method, we will apply a sensitivity analysis to this special class of zero-sum matrix games. Although we have focused on a particular version of MP, introduced first by vos Savant [11], to avoid

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1 Examples also abound in everyday life and even literature, as documented in [15], which cites passages from the works of William Faulkner, Edgar Allen Poe, and Sir Arthur Conan Doyle.

2 See [3] for a thorough analysis of the dynamics of the Matching Pennies game.

3 For example see [2].

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complication in more general cases, the method used in this paper can be easily generalized to any matrix game. In fact, this method is an effective tool to deal with practical applications of game theory. By using this method, one can realize whether he/she should or should not participate in a game, even when the parameters are random variables with known probability distributions.

Our most practical finding is the calculation of a critical value for each entry in the payoff matrix. This value determines an upper (lower) bound for the increase (decrease) in each of the payoffs so that the game favors the same player.[4]

The MP introduced by vos Savant [11] can be described as follows. "The game has two players, each one given a fair coin labeled with a head and a tail. Then, each player will show either a head or a tail. If both players show a head, then the second player (Player #2) wins $3. If both players show a tail, then Player #2 wins $1. If one player shows a head and the other shows a tail, then Player #1 wins $2". According to the Fundamental Theorem of Matrix Games, this game has at least one optimal strategy for each player which can be found by linear programming (LP).

We found the optimal strategies as well as the value of the game after changing the payoff of several outcomes in the payoff matrix. Then we compared the results with those of Jacobson and Shyryayev [12] and found a close similarity in the results, although ours were more general.

2. Formulating MP as a matrix game

According to the above-mentioned description, MP can be modeled as a zero-sum matrix game by the following payoff matrix[5]:

		Player #2: H	Player #2: T
Player #1	H	−3	2
	T	2	−1

where **H** represents a head and **T** represents a tail. Each number refers to player #1's gain. A negative number indicates a loss for player #1 or equivalently a gain for player #2, since the game is zero-sum.

3. Formulating MP as a LP problem and finding optimal strategies

Each matrix game can be formulated as a LP problem in several ways. We have used the procedure discussed in [13]:

3.1. Solving a matrix game

Imagine a zero-sum matrix game as follows:

		Column Player: c_1	c_2	…	c_m
Row Player	r_1	a_{11}	a_{12}	…	a_{1m}
	r_2	a_{21}	a_{22}	…	a_{2m}
	…	…	…	…	…
	r_n	a_{n1}	a_{n2}	…	a_{nm}

where $c_j(r_i)$ is the probability of choosing strategy $j(i)$ by the Column Player (Row Player).

First, we check for the existence of saddle points. If there is one, you can solve the game by selecting each player's optimal pure strategy. Otherwise, continue with the following steps:

Step 1: Reduce the payoff matrix by dominance.

Step 2: Add a fixed number k to each of the entries. Therefore, all entries become non-negative.

Step 3: Solve the corresponding LP problem using the simplex method.[6]

$$\textit{maximize } p = x_1 + x_2 + \cdots + x_m$$

subject to:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m \leq 1$$

4 See Section 5 for an accurate definition of the "critical value".

5 See [14] for the detailed procedure of representing a matrix game.

6 See [16] for a complete discussion of the simplex method.