International Journal of Non-Linear Mechanics 45 (2010) 121–139

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Dynamic bifurcation and sensitivity analysis of non-linear non-planar vibrations of geometrically imperfect cantilevered beams

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A R T I C L E   I N F O

Article history:
Received 14 February 2009
Received in revised form
6 October 2009
Accepted 6 October 2009

Keywords:
Bifurcation
Limit-cycle
Chaos
Sensitivity
Geometric imperfection

A B S T R A C T

The non-linear non-planar dynamic responses of a near-square cantilevered (a special case of inextensional beams) geometrically imperfect (i.e., slightly curved) and perfect beam under harmonic primary resonant base excitation with a one-to-one internal resonance is investigated. The sensitivity of limit-cycles predicted by the perfect beam model to small geometric imperfections is analyzed and the importance of taking into account the small geometric imperfections is investigated. This was carried out by assuming two different geometric imperfection shapes, fixing the corresponding frequency detuning parameters and continuation of sample limit-cycles versus the imperfection parameter. The branches of periodic responses for perfect and imperfect (i.e. small geometric imperfection) beams are determined and compared. It is shown that branches of periodic solutions associated with similar limit-cycles of the imperfect and perfect beams have a frequency shift with respect to each other and may undergo different bifurcations which results in different dynamic responses. Furthermore, the imperfect beam model predicts more dynamic attractors than the perfect one. Also, it is shown that depending on the magnitude of geometric imperfection, some of the attractors predicted by the perfect beam model may collapse. Ignoring the small geometric imperfections and applying the perfect beam model is shown to contribute to erroneous results.

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1. Introduction

The non-linear dynamic response of a long, slender beam has been the subject of many theoretical and experimental efforts due to the fact that engineering structures like helicopter rotor blades, spacecraft antennae, flexible satellites, airplane wings, gun barrels, robot arms, high-rise buildings, long-span bridges, and subsystems of more complex structures can be modeled as a beam-like slender member. Linear perfect (i.e. ignoring geometric imperfections) modeling of small-amplitude vibrating beams may contribute to results that match the experimental observations but when the amplitude of excitation is large, it does not predict the dynamic responses correctly. This is the consequence of ignoring several non-linearities such as inertia, curvature, mid-plane stretching, natural geometric imperfection and various beam effects like shear deformation, warping and rotary inertia. In the following survey, a brief summary of the most relevant works is presented.

Crespo da Silva and Glynn [1] investigated the flexural–flexural–torsional dynamics of beams to primary resonances accounting for both geometric and inertia non-linearities. They found that the first and the second mode response curves are different and response curves for higher modes are approximately independent of non-linear curvature terms. Crespo da Silva [2] used the same equations including damping and investigated the whirling motions of base-excited cantilever beams. He found that some whirling motions are unstable; furthermore, neither planar nor non-planar stable steady state motions existed in some ranges of frequency detuning. Crespo da Silva [3] investigated the planar response of an extensional beam to a periodic excitation. He found that the effect of the non-linearity due to midplane stretching is dominant and that neglecting the non-linearities due to curvature and inertia does not introduce significant error in the results. Also, unlike the response of an inextensional beam, the single-mode response of an extensional beam is always hardening. Pai and Nayfeh [4] investigated the non-planar oscillations of compact (i.e. near-square) beams under lateral base excitations. They located Hopf bifurcations and found that the system can exhibit quasi-periodic or chaotic motions; furthermore, the low-frequency modes are dominated by geometric non-linearities while the high-frequency modes are dominated by inertia non-linearities. Shyu et al. [5] used the equation in [1] to investigate the stationary whirling responses of a cantilever beam with static deflection to a subharmonic resonance of order one-half and a superharmonic resonance of order two. Restuccio et al. [6] investigated the planar and non-planar motions of the clamped–clamped/sliding beam to a principle parametric resonance when the cross-section is nearly square using the equations in [1].

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0020-7462/$ - see front matter © 2009 Published by Elsevier Ltd.
doi:10.1016/j.ijnonlinmec.2009.10.002

Nomenclature

m	mass per unit length of the beam
D_{ii}, $i=\eta,\zeta$	flexural stiffness constants
$v(x,t)$,$w(x,t)$	beam neutral axis deflections along Y and Z axes
$v_0(x)$, $w_0(x)$	beam neutral axis initial deflections along Y, Z axes
d_i, $i=v,w$	damping coefficients
F	base excitation amplitude
Ω	excitation frequency
ω_{ni}, $i=z,y$	beam nth natural frequency along Z and Y axes
σ	excitation frequency detuning parameter
δ	beam cross section out-of-squareness detuning parameter
$\eta\zeta\xi$	current principle axes coordinate system of beam cross-section
XYZ	inertial reference coordinate system

Depending on the excitation amplitude, the damping, and the ratio of the principal flexural rigidities, they uncovered various frequency-response diagrams and found periodically and chaotically modulated motions. They found that the transition from periodic motions to chaotically modulated motions is via the process of torus doubling and subsequent destruction of the torus. Arafat et al. [7] used the equations in [1] to investigate the planar and non-planar dynamics of a cantilever beam to a principal parametric resonance of one of the modes in the presence of a one-to-one internal resonance between two in-plane and out-of-plane modes. They investigated the possible bifurcations in the planar and non-planar responses when the excitation frequency is varied. They found that over some frequency interval, the non-planar periodic motions can bifurcate to amplitude- and phase-modulated motions with the modulation being either periodic or chaotic. Avramov [8] investigated the non-linear oscillations of a simply supported beam subjected to a periodic force at a combination resonance. He used the center manifold method and discovered the Naimark–Sacker bifurcations leading to almost-periodic oscillations. Dwivedy and Kar [9] investigated the non-linear dynamics of a base-excited slender beam carrying a lumped mass subjected to simultaneous combination parametric resonance of sum and difference types along with 1:3:5 internal resonances. They observed interesting phenomena like blue sky catastrophe, jump down phenomena and simultaneous occurrence of periodic and chaotic orbits. Lacarbonara et al. [10] investigated non-linear interactions in a hinged–hinged uniform moderately curved beam with a torsional spring at one end. The beam mixed-mode response is shown to undergo several bifurcations, including Hopf and homoclinic bifurcations, along with the phenomenon of frequency island generation and mode localization. Zhang [11] conducted an analysis of the chaotic motion and its control for the non-linear non-planar oscillations of a cantilever beam subjected to a harmonic axial excitation and transverse excitations at the free end. Based on the averaged equations that he obtained, numerical simulation was utilized to discover the periodic and chaotic motions for the non-linear non-planar oscillations of the cantilever beam. The numerical results indicate that the transverse excitation in the Z direction at the free end can control the chaotic motion to a period n motion or a static state for the non-linear non-planar oscillations of the cantilever beam.

Luongo and Egidio [12] applied the multiple scales method to a one-dimensional continuous model of planar, inextensible and shear-undeformable straight beam to derive the equations governing the system asymptotic dynamic around a bifurcation point. They studied the post-critical behavior around the bifurcations. Paolone et al. [13] analyzed the stability of a cantilever elastic beam with rectangular cross-section under the action of a follower tangential force and a bending conservative couple at the free end. The linear stability of the trivial equilibrium is studied, revealing the existence of buckling, flutter and double-zero critical points.

Zhang [14] investigated the multi-pulse global bifurcations and chaotic dynamics for the non-linear non-planar oscillations of a cantilever beam subjected to a harmonic axial excitation and two transverse excitations at the free end by using an extended Melnikov method in the resonant case. The results of numerical simulation show that the Shilnikov-type multi-pulse chaotic motions can occur for the non-linear non-planar oscillations of the cantilever beam, which verifies the analytical prediction.

Aghababaei et al. [15] derived the non-linear equations and boundary conditions of non-planar (two bending and one torsional) vibrations of inextensional isotropic geometrically imperfect beams (i.e. slightly curved and twisted beams) using the extended Hamilton's principle. The order of magnitude of the natural geometric imperfection was assumed to be the same as the first order of vibrations amplitude. Although the natural imperfection is small, their study shows that in contrast to the case of straight beams (i.e. geometrically perfect beams), the vibration equations are linearly coupled and have linear and quadratic terms in addition to cubic terms. Also, in the case of near-square or near-circular beams, coupling terms between lateral and torsional vibrations exist. Furthermore, a problem of parametric excitation in the case of perfect beams changes to a problem of mixed parametric and external excitation in the case of imperfect beams. They have also investigated the validity of the proposed model using the existing experimental data.

Aghababaei et al. [16] investigated the non-linear non-planar steady-state responses of a near-square cantilevered beam (a special case of inextensional beams) with geometric imperfection under harmonic base excitation using the equations in [15]. By applying the combination of the multiple scales method and the Galerkin procedure to two non-linear integro-differential equations derived in [15], two modulation non-linear coupled first-order differential equations were obtained for the case of a primary resonance with a one-to-one internal resonance. They showed that the modulation equations contain linear imperfection-induced terms in addition to cubic geometric and inertial terms. Variations of the steady-state response amplitude curves with different parameters were presented. Bifurcation analyses of fixed points show that the influence of geometric imperfection on the steady-state responses can be significant to a great extent although the imperfection is small. The phenomenon of frequency island generation was also observed.

In this paper, first the limit-cycles of the perfect cantilever beam are determined. Then, by assuming two different geometric imperfection shapes, two sample determined limit-cycles are continued versus the imperfection parameter (this is done by assuming that the corresponding frequency detuning parameter is fixed) and the sensitivity of those limit-cycles to small geometric imperfections is investigated and the importance of taking into account the small geometric imperfections is discussed for the first time. Then, by invoking a special geometric imperfection shape, the limit-cycles of the imperfect beam are computed by incorporating the imperfect beam model. By continuation of the computed limit-cycles for the imperfect beam and those for the perfect beam, the branches of dynamic solutions for both beams are determined and compared. The effect of small geometric imperfections on the dynamic bifurcations is extensively studied.

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