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Sensitivity analysis for the design of profile extrusion dies

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ABSTRACT

This paper proposes a procedure for design sensitivity analysis using the direct differentiation method which can be easily included into a finite element code. By a reformulation of the governing equations, it is shown that the derivatives necessary to evaluate the sensitivity of a given performance measure corresponding to the current design configuration can be obtained through a post-processing step by the same finite element solver used to solve the discrete state equations. The procedure is illustrated for some examples with known analytical expression of the sensitivity and then applied to several practical problems in the design of an extrusion process.

1. Introduction

Extrusion is the process used to manufacture products in the form of continuous lengths with a uniform cross section. It is the task of the die to convert the cylindrical flow from the extruder into the required cross section. The design of a profile die is consequently a formidable task due to the intricate cross sections and tight dimensional tolerances that are usually required, coupled with the complexity of the flow phenomena involved in extrusion.

The significant advances of the modelling techniques during the recent years have made possible the simulation of the complex physical phenomena occurring in manufacturing processes. By now, there are several commercial and research software tools that have been developed for these simulations [1]. By including in such codes also the facility of the design sensitivity analysis, one can obtain an efficient design methodology.

A sensitivity analysis, which is the subject of the current paper, aims to quantitatively estimate the importance of a given change of parameters or design variables with respect to a global die design, without requiring trial and error attempts. It also represents a step in gradient-based optimization algorithms [2–5].

Progress in sensitivity analysis described for instance in [5–9], have made it feasible to use intensive numerical methods in design optimization for real applications in general [10,11,3] and in material processing designs such as polymer extrusion [10,12–14], metal forming processes [15], die shape design in sheet metal stamping processes [7], and polymer injection and compression moulding processes [12,16], in particular.

In mathematical terms, the objective of a sensitivity analysis is the evaluation of the derivative of a given performance measure with respect to the design variables for given admissible configurations. The performance measure quantifies a certain process behaviour that one means to monitor. It is generally expressed as functional of the design variables and of the response or state variables. Since the state variables generally depend on the design variables through the state or equilibrium equations, the main concern in the sensitivity analysis is the evaluation of the implicit variations of these variables. The most common and used methods that are found in the literature are [6,17]: the finite difference method (FDM); the adjoint variable method (AVM); and the direct differentiation method (DDM). FDM represents the most straightforward way to compute sensitivities: a small design perturbation is introduced for each design variables, and the gradient of the response is then computed using a finite difference scheme to approximate the derivative. FDM is easy to implement although computationally expensive, since it requires that the problem is solved again for the perturbed value of each of the design variable. This method will however be considered in this paper to assess the proposed procedure. The AVM uses the solution to the adjoint problem to eliminate the implicit derivations that appear in the performance measure; its use is convenient when there are many performance measures to be considered. However, for the case of nonlinear problems, the AVM applies to a linearization version. DDM uses implicit differentiation to differentiate the state variables, involving the differentiation of the governing equations, and can be applied also to nonlinear problems.

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An important aspect in the design problem of an extrusion process concerns the geometry change. In such a case the governing equation will be defined in a variable domain, and therefore parameters that control the geometry of the body need to be included in the set of design variables. The domain parameterization method [18,19] and the material derivative method [20–22] are alternative techniques to represent the shape variations. The former method defines the derivatives assuming a fixed reference domain and applies the chain rule. A transformation that links the fixed reference domain, i.e. an invariant geometry, with the material coordinates is also inherent to FEM implementation where element coordinates serve as reference coordinates and the shape design variation can be represented by variations of the nodal coordinates, with the shape functions held fixed [17,23]. The material derivative method defines the derivatives at the undeformed (or initial) configuration instead, and depends on the domain design velocity, which represents the direction of a given design perturbation.

In this paper we are concerned with the inclusion of a sensitivity analysis approach into a finite element (FE) code using the DDM. Unlike current methods for sensitivity analysis and in order to keep the modifications to the FE code at a minimum, we develop a single procedure that yields the solution, the sensitivity values and the performance measure values within the same iterative strategy employed to solve the governing system of equations. The details of the sensitivity formulations are fully consistent with the details of the numerical model and the solution algorithm used in the simulation. This means that identical spatial discretization, interpolation functions and degrees of freedom are considered in the simulation and the design derivatives of the state variables are all well defined [17]. Highly accurate sensitivity predictions can therefore be obtained with the proposed procedure. The striking feature of our procedure is that it can be easily implemented in existing finite element codes without changes to the core part of the codes, i.e. the solver and the data structure.

With the aim of providing a powerful, flexible design aid for interactive use by the designer, the proposed sensitivity analysis approach is implemented into the commercial, finite element POLYFLOW [24] solver and included into an extrusion die design environment. This design environment is based on the classic three-column concept [25] and consists of: the structural model; the optimization algorithm, and the (design) optimization model.

The remainder of the paper is organized as follows: in Section 2 we introduce a consistent sensitivity analysis formulation for a steady state problem, formulated for treating the case of change of shapes using the domain parameterization method. In Section 3 we propose a numerical procedure to evaluate the sensitivity of a performance measure for any given design configuration within the same iterative strategy used to solve the governing system of equations. The proposed procedure is then developed in full details for two classical analytical examples in Section 4 with the expressions of the augmented matrices given in the Appendix. The procedure is then applied to the design of extrusion dies in Section 5. In Section 6 we validate the procedure by applying it to the design of two industrial dies. This is followed by the conclusions in Section 7.

2. Sensitivity analysis

This section introduces the design sensitivity analysis for the steady state response of a discrete system obtained by applying the FE method. We denote by $\boldsymbol{b}^T = [b^1, \ldots, b^k] \in R^k$ the vector of design or control variables. The vector $\boldsymbol{b}$ can include shape and/or non-geometric variables, such as material properties. The domain parametrization method is used to represent the design geometry [18], and the direct differentiation method is adopted to evaluate the derivative of the implicitly defined variables.

2.1. Statement of the problem

Let us denote by F: $R^k \times R^{m\times d} \times R^{m\times d} \to R$ a scalar function which represents a *performance measure*, such as, weight of the structure, displacement at a point, mean stress in a certain region, pressure drop across a die, exit velocities, and define

$$f(\boldsymbol{b}) := F(\boldsymbol{b}, \boldsymbol{x}(\boldsymbol{b}), \boldsymbol{u}(\boldsymbol{b})) \tag{1}$$

with $\boldsymbol{x} \in R^{m\times d}$ for $m = 1,2,3$ denoting the nodal coordinates of the FE mesh , d the number of nodes, and $\boldsymbol{u} \in R^{m\times d}$ the discrete solution of the state equation. We assume that for a given $\boldsymbol{b}$, there exists a mapping $\boldsymbol{R}_X$: $R^k \times R^{m\times d} \to R^{m\times d}$ such that $\boldsymbol{x}$ is defined implicitly in terms of $\boldsymbol{b}$ by the following equation

$$\boldsymbol{R}_X(\boldsymbol{b}, \boldsymbol{x}(\boldsymbol{b})) = 0 \tag{2}$$

Also, the mapping $\boldsymbol{R}_U$: $R^k \times R^{m\times d} \times R^{m\times d} \to R^{m\times d}$ is introduced to represent the discrete form of the state equations as follows

$$\boldsymbol{R}_U(\boldsymbol{b}, \boldsymbol{x}(\boldsymbol{b}), \boldsymbol{u}(\boldsymbol{b})) = 0 \tag{3}$$

2.2. Design sensitivity analysis

The sensitivity analysis aims to quantify the change in value of the assigned performance measure for a given variation of the design variable. This amounts to compute the derivative of f with respect to $\boldsymbol{b}$. Assuming that F, together with $\boldsymbol{R}_X$ and $\boldsymbol{R}_U$, enjoy all the regularity properties we need, and using the chain rule we have

$$\frac{df}{d\boldsymbol{b}} = \frac{\partial F}{\partial \boldsymbol{b}} + \frac{\partial F}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}} + \frac{\partial F}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}} \tag{4}$$

In Eq. (4), the derivatives $\frac{\partial F}{\partial \boldsymbol{b}}$, $\frac{\partial F}{\partial \boldsymbol{x}}$ and $\frac{\partial F}{\partial \boldsymbol{u}}$ can in general easily be evaluated, given the explicit dependence of F on $\boldsymbol{x}$, $\boldsymbol{u}$ and $\boldsymbol{b}$; whereas the evaluation of $\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}}$ and $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}}$ offers major difficulties since the variables $\boldsymbol{x}$ and $\boldsymbol{u}$ are implicit functions of $\boldsymbol{b}$. Differentiating Eqs. (2) and (3) yields

$$\begin{aligned} \frac{d\boldsymbol{R}_X}{d\boldsymbol{b}} &= 0 = \frac{\partial \boldsymbol{R}_X}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{R}_X}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}} \\ \frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{b}} &= 0 = \frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}} \end{aligned} \tag{5}$$

that can be rewritten as follows:

$$\begin{aligned} &\frac{\partial \boldsymbol{R}_X}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}} = -\frac{\partial \boldsymbol{R}_X}{\partial \boldsymbol{b}}, \\ &\frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}} = -\left(\frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}}\right) \end{aligned} \tag{6}$$

and must be solved with respect to the components of the matrices $\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}}$ and $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}}$.

Remark. Notice that $\frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{u}}$ is a matrix $(m \times d) \times (m \times d)$ and $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}}$ is a matrix $(m \times d) \times k$, hence $\frac{\partial \boldsymbol{R}_U}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{b}}$ denotes the standard row by column product between matrices. The same observation applies for the other terms.

3. Numerical procedure

In this section we show that for any given design configuration the sensitivity of f can be evaluated within the same iterative strategy used to solve the nonlinear equations (2) and (3). Let $\bar{b}ib$ denote the value of the design variables defining the given configuration, then Eqs. (1)–(3) can be equivalently written as follows:

$$\begin{aligned} &\text{Find } [\boldsymbol{b}, \boldsymbol{x}, \boldsymbol{u}, f] \text{ such that :} \\ &\boldsymbol{R_b}(\boldsymbol{b}) := \boldsymbol{b} - \bar{\boldsymbol{b}} = \boldsymbol{0} \\ &\boldsymbol{R}_X(\boldsymbol{b}, \boldsymbol{x}) = \boldsymbol{0} \\ &\boldsymbol{R}_U(\boldsymbol{b}, \boldsymbol{x}, \boldsymbol{u}) = \boldsymbol{0} \\ &R_F(\boldsymbol{b}, \boldsymbol{x}, \boldsymbol{u}, f) := f - F(\boldsymbol{b}, \boldsymbol{x}, \boldsymbol{u}) = 0 \end{aligned} \tag{7}$$