



## Sensitivity analysis of critical forces of trusses with side bracing

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### ARTICLE INFO

#### Article history:

Received 20 June 2009

Accepted 11 February 2010

#### Keywords:

Truss  
Lateral buckling  
Bracing  
Effective length

### ABSTRACT

The present research is devoted to the study of out-of-plane buckling of trusses with elastic side bracing. In this paper, a sensitivity analysis of critical buckling loads of a truss due to bracing stiffness is carried out. A method based on the sensitivity analysis for the determination of the threshold bracing stiffness condition for full bracing of a truss is proposed. The influence lines of the unit change of the bracing stiffness on the buckling load, for different initial bracing stiffness, are investigated. The approximations of an exact relation between the buckling load and bracing stiffness are found. The buckling length related to the side-support distance as a function of bracing stiffness is also determined. It is shown that the buckling length of truss chords with elastic side supports is larger than that assumed in design codes.

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### 1. Introduction

Steel trusses have a much greater strength and stiffness in their plane than out of their plane, and therefore should be braced against lateral deflection and twisting. The problem of bracing requirements necessary to provide lateral stability of compressed members is present in the Polish design codes [1], and in the Eurocode [2]. The simplified design code requirements allow one to assume that the buckling length of truss chords is equal to the distance between braces. In this approach, only the truss top chord is considered. The effect of the lower chord, verticals and diagonals on the truss stability is neglected. Verticals and diagonals are considered only as vertical supports to the upper truss chord, side bracing of the truss chords is considered as a rigid side support, and normal forces in the truss chords are assumed to be constant along their length. Under the above conditions, the buckling length of compressed truss chords is usually lower than it is described as being in the codes. Such a result was obtained by Biegus and Wojcyszyn [3].

However, the bracing is usually elastic, and other truss elements such as compressed diagonals or verticals may buckle locally, and this may lower the critical loading of the truss.

Many solutions of restrained column and beam buckling are presented in the literature (see, for example Trahair [4]). Most code requirements concerning bracing are based on the principles formulated by Winter [5], who introduced a simple model with fictitious hinges at the braced joints. His research was focused on an estimation of the safe lower limit of the necessary rigidity of the

bracing, such that the braced element would attain maximal critical force and that buckling occurs between braces. In research conducted by Yura [6], the bracing requirement was extended to cases where the bracing stiffness is less than the full bracing condition. Bracing requirements for frame structures were investigated by Tong and Ji [7], where a threshold bracing condition for full bracing of plane frames was derived and a critical buckling force for weakly braced frames was determined. Studies conducted by Girgin, Özmen and Orakdogan [8] showed that code formulae for determining the buckling lengths of frame columns may yield erroneous results, especially for irregular frames.

Similar problems of determining the bracing requirements of trusses are present in only a few studies. The stability of trusses with elastic bracing was investigated in experimental research by Kołodziej and Jankowska-Sandberg [9] and verified by numerical analysis [10] by Iwicki. The results of the author's numerical studies [11] of two roof trusses with horizontal and sloping elastic bracing have shown that the buckling length of truss compressed chords is greater than the side-support spacing. The spatial stability of trusses designed according to the Polish code [1] is provided even when the buckling length of truss chords is greater than the side-support spacing.

The stability of a truss with both linear and rotational bracing was analyzed by Iwicki [12]. For a truss examined with both linear and rotational springs, the limit normal force in the chords was between 20% and 70% greater than in the case without rotational springs.

The present research is focused on the determination of the full bracing condition for a truss with elastic bracing. The basic problem under consideration is devoted to investigating the required bracing stiffness that ensures that the out of truss plane buckling occurs between braces, or is prevented, so the buckling occurs in the plane of the truss. The full bracing condition may also be

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defined as the bracing stiffness that causes the maximal buckling load of the truss, or when an increase in bracing stiffness does not result in a further increase in the buckling load.

In the study, the sensitivity analysis method developed by Haug, Choi, and Komkov [13] is used. This method enables one to obtain the influence lines of the buckling load variations due to unit changes in the bracing stiffness. The approximations of an exact relation between the buckling load and bracing stiffness are found. For different stiffnesses of elastic side supports, the critical load and coefficient of buckling length of the truss chord are calculated. The results are compared to established solutions presented by Trahair [4] and Winter [5]. The sensitivity of truss stability was also analyzed in [11] by Iwicki, but that analysis was confined to sensitivities related to the non-linear limit load of imperfect trusses.

## 2. Sensitivity analysis of critical forces due to bracing stiffness

The subject of the sensitivity analysis method is to find a relation between variations of design variables and variations of structural response. The bracing stiffness may be regarded as a design variable and the critical forces may be considered a structural response. The equilibrium equation for a discrete structural system can be written as

$$(\mathbf{K} - P\mathbf{K}_G)\mathbf{z} = \mathbf{0} \quad (1)$$

where  $\mathbf{K}$  is the stiffness matrix,  $\mathbf{K}_G$  is the geometrical matrix,  $\mathbf{z}$  denotes the nodal displacement vector and  $P$  is the critical load multiplier. Assuming that the displacement vector is normalized with respect to the geometrical matrix by the condition

$$\mathbf{z}^T \mathbf{K}_G \mathbf{z} = 1 \quad (2)$$

and that the global stiffness and geometric matrices are positive definite and differentiable with respect to a vector  $\mathbf{u}$  of design variables, we can differentiate Eq. (1) with respect to the design variables  $\mathbf{u}$

$$\mathbf{z}^T \mathbf{K}_{,u} \mathbf{z} + \mathbf{z}^T \mathbf{K} \mathbf{z}_{,u} = \mathbf{z}^T P_{,u} \mathbf{K}_G \mathbf{z} + \mathbf{z}^T P \mathbf{K}_{G,u} \mathbf{z} + \mathbf{z}^T P \mathbf{K}_G \mathbf{z}_{,u}. \quad (3)$$

Using the normalization condition (2), we can obtain the first derivative of the buckling load with respect to the design variables in the form

$$P_{,u} = \mathbf{z}^T (\mathbf{K}_{,u} - P\mathbf{K}_{G,u}) \mathbf{z} + \mathbf{z}^T (\mathbf{K} - P\mathbf{K}_G) \mathbf{z}_{,u}. \quad (4)$$

The second term in the above equation is zero because the structure must hold the equilibrium condition. Since we have the first derivative of the buckling load with respect to an arbitrary design variable, we can obtain an equation for the first variation of critical load with respect to variation of the vector of design variables in the following form

$$\delta P_{cr} = P_{cr,u} \delta \mathbf{u} = \mathbf{z}^T (\mathbf{K}_{,u} - P\mathbf{K}_{G,u}) \mathbf{z} \delta \mathbf{u} = \Lambda_{P_{cr,u}} \delta \mathbf{u} \quad (5)$$

where the vector  $\Lambda_{P_{cr,u}}$  describes the influence of the unit change of the design variable on the buckling load. Eq. (5) may be used to calculate the first variation of critical buckling force caused by the variation of bracing stiffness. The vector  $\Lambda_{P_{cr,u}}$  allows one to determine which parts of the structure, when applying a bracing, may result in the largest variation of the buckling load. Eq. (5) may easily be applied to most commercial structural analysis programs such as ROBOT STRUCTURAL ANALYSIS PROFESSIONAL [14]. In order to calculate the variations of critical forces due to linear bracing variations, the buckling mode normalized according to condition (5) is needed. When the bracing is located at nodes, the derivative in brackets in Eq. (5) is equal to one. The calculation of the first variation of critical forces may be conducted by means of a commercial spreadsheet program such as EXCEL [15].

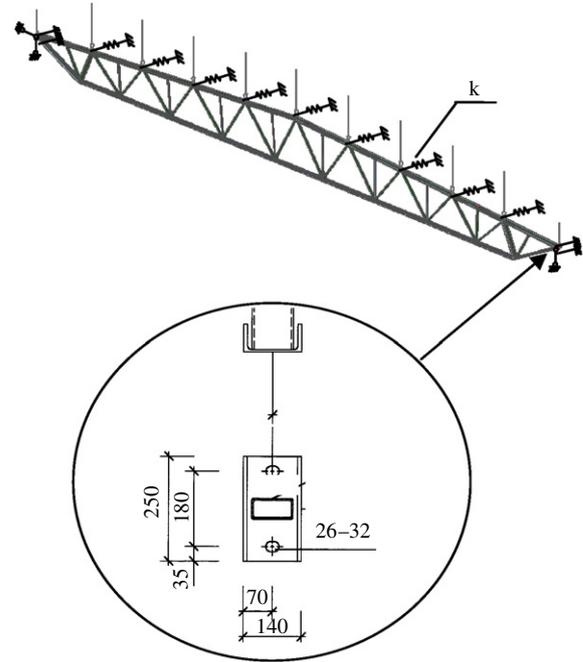


Fig. 1. Truss with linear elastic bracing.

## 3. Model description

In the present parametric study, a roof truss as shown in Fig. 1 is considered. The depth of the truss is 1.61 m at midspan and 0.9 m at the supports. This truss is torsionally relatively weak, because the only torsional restraint is at the supports and it consists of two construction bolts spaced at 0.18 m that restrain the truss against twisting at the supports. The compression chord consists of 2L 90 × 90 × 9, the tension chord is made of 2L 80 × 80 × 8 rolled profiles. Two compression diagonals near the supports are made of 2L 65 × 65 × 7 profiles. Other diagonals are made of U65 profiles. The connections between the truss chord, diagonal, and vertical elements are rigid, so the bottom chord, diagonals and verticals interact together with the truss top chord and partially restrain the top chord against out-of-plane buckling. It is assumed that the load is applied as nine concentrated forces of 25 kN at the top chord joints, and its magnitude represents the dead load and snow load acting on the roof structure. The top chord is laterally braced at the joints by linear elastic side supports spaced 2.4 m apart. The built-up top chord section is batted every 0.6 m to avoid the buckling of individual members. The batten consists of U65 profile and is located between profiles of the truss top chord. The chord out-of-plane buckling force is 4465.41 kN at a buckling length of 2.4 m, while the buckling force of the chord in the plane of the truss is 3259.71 kN at a buckling length of 1.2 m.

The stability analysis of the 3D truss model was carried out by means of the ROBOT STRUCTURAL ANALYSIS PROFESSIONAL program [14]. Spatial beam elements with six degrees of freedom at each node were used to model the truss and linear springs were used to model the side supports.

## 4. Results of numerical simulations

### 4.1. First variation of critical buckling load due to variation of bracing stiffness

The first critical buckling load variation due to the variation of bracing stiffness was calculated by means of Eq. (5). Several different initial bracing stiffnesses were considered. The influence

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