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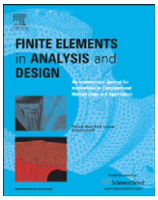

Nonlinear sensitivity analysis of reinforced concrete frames

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ABSTRACT

Design sensitivity analysis is a necessary task for design optimization of structures. Methods of sensitivity analysis for linear systems have been developed and well documented in the literature; however there are a few such research works for nonlinear systems. Nonlinear sensitivity analysis of structures under seismic loading is very complicated. This paper presents an analytical sensitivity technique for reinforcement concrete moment resisting frames (RCMRF) that accounts for both material nonlinearity and geometric effects under pushover analysis. The results of the proposed method are compared with the results of finite difference method (FDM). Two examples including one three-story, two bays moment frame and one ten-story, two-bay frame are used to illustrate the efficiency and accuracy of the method and difficulties of the FDM for nonlinear sensitivity analysis (NSA) of RCMRF are discussed. The proposed technique can be very useful and efficient for optimal performance-based design of RC buildings.

1. Introduction

Structural optimization for linear response is a well-defined problem and a large number of research works have been carried out on this subject. In recent years, performance-based seismic design has become a necessity for design of new structures. Generally pushover analysis, which is a simplified static nonlinear procedure, is used for performance-based design [22]. In the pushover analysis a predefined pattern of earthquake loads is applied incrementally on the structure until a predefined target displacement is reached or a plastic collapse mechanism is occurred [1,11]. Accordingly for performance-based design, structural optimization involves in nonlinear analysis.

Structural design optimization involves response sensitivity analysis for formulating constraint functions in their explicit form [25]. Considerable part of computational effort in an optimization problem is usually allocated to the sensitivity analysis. Each sensitivity coefficient defines the amount of change in a structural response due to a unit change in a design variable, such as sensitivity of displacement to change in cross-sectional dimensions or reinforcement ratio.

While considerable research effort has been put on developing the sensitivity analysis techniques, there are a few research works in the literature that has focused on the theory of sensitivity analysis for nonlinear structural systems. Ryu et al. [36] proposed a general nonlinear sensitivity analysis accounting for geometric and material nonlinearity. They used modified Newton–Raphson method for their nonlinear analysis and used the same procedure for their sensitivity analysis. They presented the formulation for a truss example. Choi and Santos [8] developed variational formulations for nonlinear design sensitivity analysis. They used linearized equilibrium equations to obtain first variations of the governing nonlinear equilibrium equations with respect to design variables. Gopalakrishna and Greimann [13] differentiated the equilibrium equation in each Newton–Raphson iteration to obtain incremental gradients and nonlinear sensitivity analysis of the planar trusses. Park and Choi [34] developed a continuum formulation for sizing design sensitivity analysis of the critical load factor considering both material and geometric nonlinearity effects. They presented several truss examples for verification of their proposed method. Tsay and Arora [41] presented a general theory for sensitivity analysis of history-dependent problems considering material and geometric nonlinearity based on total Lagrangian concept using direct differentiation method (DDM) and adjoint variable method (AVM). Tsay et al. [42] verified this theory using a beam and a rod element at various cases. Santos and Choi [38] presented a unified approach for shape sensitivity analysis of trusses and beams accounting for both geometric and material nonlinearities. They utilized the adjoint variable and direct differentiation methods. Cardoso and Arora [6] presented a unified variational theory for nonlinear dynamic sensitivity analysis. Vidal and Haber [43] discussed the accuracy of the sensitivity coefficients with the reduced form of tangent operator for rate-independent elastoplasticity. Kleiber [21] employed the

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DDM and the AVM methods for sensitivity analysis accounting for material and kinematic nonlinearity. Ohsaki and Arora [28] presented an accumulative and incremental algorithm for the design sensitivity analysis of elastoplastic structures including geometrical nonlinearity. They performed the sensitivity analysis of trusses but they reported that the method is extremely time consuming for large structures. Michaleris et al. [24] derived tangent operators and design sensitivities for transient nonlinear coupled problems with elastoplasticity. Lee and Arora [23] investigated the effect of discontinuity in yield surface on the sensitivity analysis and presented a procedure for their treatments. They developed design sensitivity analysis of structural systems having elastoplastic material behavior using the continuum formulation and illustrated the sensitivity analyses for a truss and a plate by this technique. Chattopadhyay and Guo [7] developed a DDM method for nonlinear sensitivity analysis of structures undergoing elastoplastic deformation. Barthold and Stein [2] presented a continuum mechanical-based formulation for the variational sensitivity analysis accounting for nonlinear hyperelastic material behavior using either the Lagrangian or Eulerian description. Szewczyk and Ahmed [40] presented a hybrid numerical/-neurocomputing strategy for evaluation of sensitivity coefficients of composite panels subjected to combined thermal and mechanical loads. They pointed out that this method reduces the number of full-system analysis. Dimitri et al. [10] examined the influence of uncertainties on reliability of RC structures via sensitivity analysis. Yamazaki [44] suggested a direct sensitivity analysis technique for finding incremental sensitivities of the path-dependent nonlinear problem based on the updated Lagrangian formulation. Employing this method they performed the sensitivity analysis of a plate. Bugeda et al. [5] proposed a direct formulation for computing the structural shape sensitivity analysis with a nonlinear constitutive material model. It was reported that their proposed approach was valid for some specific nonlinear material models. Schwarz and Ramm [39] proposed the variational direct method for sensitivity analysis of structural response accounting for geometrical and material nonlinearity with Prantel-Reuss plasticity model. Kim et al. [20] developed an analytical method for nonlinear shape sensitivity analysis of finite deformation elastoplastic structures and used the same tangent stiffness as that of the response analysis that could speed up the computation of sensitivity. Jung and Cho [18] employed AVM method for sensitivity analysis for reliability-based topology optimization of geometrically nonlinear plate structures. Ohsaki [29] discussed on difficulties of geometrically nonlinear sensitivity analysis and critical states in trusses. Pajot and Maute [30] introduced an analytical method for geometrically nonlinear sensitivity analysis of plates. Haukaas and Scott [17] developed a unified shape sensitivity equation by DDM method considering inelastic beam–column response. For verification of the method, they performed reliability analysis for a steel frame and a RC frame. Gong et al. [12] presented a procedure for sensitivity analysis of planar steel moment frameworks accounting for geometric and material nonlinearity. In their work, analytical formulations defining the sensitivity of displacement were derived. They used the lumped plasticity model and the incremental nonlinear method for pushover analysis. They employed force-control method for evaluation of target displacement sensitivity.

While many valuable researches have been carried out for nonlinear sensitivity analysis of steel trusses, single beams, single plates and shells, etc., there are a few researches on nonlinear sensitivity analysis of RC frames. The objective of this study is to develop a comprehensive formulation for sensitivity analysis of planar RCMRF accounting for both material nonlinearity and P–Δ effects under pushover analysis using Newton–Raphson method. In this paper attention has been focused on some aspects of nonlinear sensitivity analysis that have not been previously considered in the literature. Among these aspects is the sensitivity analysis of nonlinear RC frames considering variable/seismic loading, cracking of the members and spread plasticity. Because of different material behavior of steel and concrete and cracking of the concrete, different material nonlinearity assumptions should be considered. In this study the material nonlinearity is concerned with the cracked regions of RC elements. Since the lumped plasticity model can lead to inaccurate nonlinear responses of RC frames, a spread plasticity model that is more suitable than the lumped plasticity model for RC elements has been considered. Because the moment-curvature relation of RC elements is more complex than that of steel elements and the discontinuities in moment-curvature relation of RC element may lead to inaccurate response sensitivities in the FDM method, in this research the analytical method of sensitivity analysis has been followed. The proposed procedure can be efficiently used for optimal performance-based design of RC frameworks. Two examples including a three-story and a ten-story RCMRF have been used to illustrate the applicability and efficiency of the developed sensitivity formulations.

2. Pushover analysis of RCMRF

The first step in design optimization is to calculate the sensitivities of the response of structure to change in design variables that in turn depends on the method of structural analysis. The simplest recommended method for nonlinear static analysis is pushover method. This method of analysis that is recommended by FEMA273 [11] and ATC40 [1] is a popular tool for evaluation of seismic performance of structures. Many researchers such as Saiidi and Sozen [37], Bracci et al. [4], Kilar and Fajfar [19], Gupta and Krawinkler [14], Mwafy and Elnashai [27], Hassan et al. [16] and Chopra and Goel [9] have used this analysis method in their research works. In the pushover analysis, it is necessary to specify a proper material behavior model for elements. This is explained in the next two sections.

2.1. Moment-curvature relation

The moment-curvature relation of every RC structural element has a definitive effect on the behavior of the structure. In this research the tri-linear moment-curvature relation, as shown in Fig. 1, is used for expressing the nonlinear behavior of reinforced

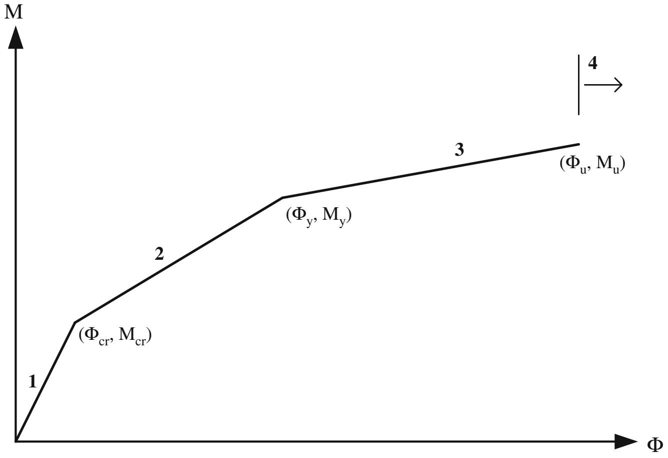

Fig. 1. Tri-linear moment curvature curve.