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Sensitivity analysis for a system of generalized mixed implicit equilibrium problems in uniformly smooth Banach spaces[☆]

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ABSTRACT

In this paper, a new system of parametric generalized mixed implicit equilibrium problems involving non-monotone set-valued mappings in real Banach spaces is introduced and studied. We first generalize the notion of the Yosida approximation in Hilbert spaces introduced by Moudafi to reflexive Banach spaces. Further, by using the notion of the Yosida approximation, we consider a system of parametric generalized Wiener–Hopf equation problems and show its equivalence to the system of parametric generalized mixed implicit equilibrium problems. By using a fixed point formulation of the system of parametric generalized Wiener–Hopf equation problems, we study the behavior and sensitivity analysis of a solution set of the system of parametric generalized mixed implicit equilibrium problems. We prove that, under suitable assumptions, the solution set of the system of parametric generalized mixed implicit equilibrium problems is nonempty, closed and Lipschitz continuous with respect to the parameters. Our results are new, and improve and generalize some known results in this field.

1. Introduction

In recent years, much attention has been devoted to developing general methods for the sensitivity analysis of solution sets of various variational inequalities, variational inclusions and equilibrium problems. From the mathematical and engineering points of view, sensitivity properties of various variational inequalities, variational inclusions and equilibrium problems can provide new insight concerning the problem being studied and can stimulate ideas for solving these problems. The sensitivity analysis of solution sets for parametric variational inequalities has been studied extensively by many authors using quite different methods. By using the projection technique, Dafermos [1], Mukherjee and Verma [2], Noor [3], Yen [4] and Verma [5] dealt with the sensitivity analysis for variational inequalities (inclusions) with single-valued mappings. By using the implicit function approach that makes use of so-called normal mappings, Robinson [6] dealt with the sensitivity analysis for variational inequalities with single-valued mappings in finite-dimensional spaces. By using the resolvent operator technique, Adly [7], Noor and Noor [8], and Agarwal et al. [9] studied the sensitivity analysis for quasi-variational inclusions with single-valued mappings. By using the projection technique and the properties of fixed point sets of set-valued contractive mappings, Ding and Luo [10] studied the behavior and sensitivity analysis of solution sets for generalized quasi-variational inequalities. Recently, Liu et al. [11], Salahuddin [12], Park and Jeong [13], Ding [14–16] and Peng and Long [17] studied the behavior and sensitivity analysis of solution sets of generalized nonlinear implicit quasi-variational inclusions of several type with set-valued mappings. Agarwal et al. [18] studied the sensitivity analysis for a system of nonlinear mixed quasi-variational inclusions with single-valued mappings and maximal monotone mappings. Ding and Yao [19] and Ding

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and Wang [20] studied the sensitivity analysis of solution sets for systems of parametric mixed quasi-variational inclusions with set-valued mappings. It is worth mentioning that most of the results in this direction have been obtained in the setting of Hilbert spaces. Recently, Kazmi and Khan [21] studied the sensitivity analysis for parametric generalized implicit quasi-variational-like inclusions involving P–η-accretive mappings in uniformly smooth Banach spaces. By using the Yosida approximation and the Wiener–Hopf equation technique, Moudafi [22] and Huang et al. [23] studied the sensitivity analysis of solutions for generalized mixed implicit equilibrium problems in Hilbert spaces.

Inspired and motivated by the above works, in this paper, we introduce and study a new system of parametric generalized mixed implicit equilibrium problems involving non-monotone set-valued mappings in real Banach spaces, which includes the system of generalized implicit variational inequalities and the system of generalized implicit variational inclusions as special cases. We first generalize the notion of the Yosida approximation in Hilbert spaces introduced by Moudafi [22] to reflexive Banach spaces. By using the notion of the Yosida approximation, we consider a system of parametric generalized Wiener–Hopf equation problems and show its equivalence to the system of parametric generalized mixed implicit equilibrium problems. Using a fixed point formulation of the system of parametric generalized Wiener–Hopf equation problems, we study the behavior and sensitivity analysis of solution sets of the system of parametric generalized mixed implicit equilibrium problems in uniformly smooth Banach spaces. We prove that, under suitable assumptions, the solution set of the system of parametric generalized mixed implicit equilibrium problems is nonempty, closed and Lipschitz continuous with respect to the parameters. Our results are new, and improve and generalize some known results in the field.

2. Preliminaries

Let B be a real Banach space with norm $\|\cdot\|$ and let B^* be its dual space. Let $\langle\varphi, x\rangle$ denote the duality pairing between B^* and B, where $\varphi \in B^*$ and $x \in B$. Let K be a nonempty, closed and convex subset of B and let $C(B)$ be the family of all nonempty compact subsets of B.

Definition 2.1. Let K be a closed convex subset of a Hausdorff topological vector space E. A real-valued bifunction $F : K \times K \to (-\infty, \infty)$ is said to be:

(1) monotone if

$$F(x, y) + F(y, x) \leq 0 \quad \forall x, \ y \in K;$$

(2) strictly monotone if

$$F(x, y) + F(y, x) < 0 \quad \forall x, \ y \in K \text{ with } x \neq y;$$

(3) α-strongly monotone if there exists a $\alpha > 0$ such that

$$F(x, y) + F(y, x) \leq -\alpha\|x - y\| \quad \forall x, \ y \in K;$$

(4) upper hemicontinuous if

$$\limsup_{t\to 0} F(tz + (1 - t)x, y) \leq F(x, y) \quad \forall x, \ y, z \in K.$$

Remark 2.1. Clearly the strong monotonicity of F implies the monotonicity of F.

Definition 2.2. A mapping $\eta : B \times B \to B^*$ is said to be:

(i) monotone if

$$\langle\eta(x, y), x - y\rangle \geq 0, \quad \forall x, y \in B;$$

(ii) δ-strongly monotone if there exists a $\delta > 0$ such that

$$\langle\eta(x, y), x - y\rangle \geq \delta\|x - y\|^2, \quad \forall x, y \in B;$$

(iii) affine in the second argument if

$$\eta(y, \beta x + (1 - \beta)z) = \beta\eta(y, x) + (1 - \beta)\eta(y, z), \quad \forall\beta \in [0, 1], \ x, y, z \in K;$$

(iv) τ-Lipschitz continuous if there exists a constant $\tau > 0$ such that

$$\|\eta(x, y)\| \leq \tau\|x - y\|, \quad \forall x, y \in H.$$

Remark 2.2. If $B = H$ is a Hilbert space, then the concept (ii) and (iv) reduces to the corresponding concept in [23,24].

The following result is a special case of Theorem 3.9.3 of Chang [25].