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Design sensitivity analysis and optimization of interface shape for zoned-inhomogeneous thermal conduction problems using boundary integral formulation

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ABSTRACT

A generalized formulation of the shape design sensitivity analysis for two-dimensional steady-state thermal conduction problem as applied to zoned-inhomogeneous solids is presented using the boundary integral and the adjoint variable method. Shape variation of the external and zone-interface boundary is considered. Through an analytical example, it is proved that the derived sensitivity formula coincides with the analytic solution. In numerical implementation, the primal and adjoint problems are solved by the boundary element method. Shape sensitivity is numerically analyzed for a compound cylinder, a thermal diffuser and a cooling fin problem, and its accuracy is compared with that by numerical differentiation. The sensitivity formula derived is incorporated to a nonlinear programming algorithm and optimum shapes are found for the thermal diffuser and the cooling fin problem.

1. Introduction

The shape optimization problem is to find a continuum shape minimizing an objective function under prescribed constraints. The shape design sensitivity analysis (SDSA) is required to predict the sensitivity of state variables or performance functionals in numerical implementation of shape optimization algorithms. Accurate prediction of the shape sensitivity is very important because inaccurate sensitivity may cause much computational time and moreover lead to undesirable solution in numerical optimization. Hence, the SDSA has been one of intensive subjects to researchers of the shape optimization.

Nowadays, the boundary element method (BEM) or the boundary integral equation (BIE) has especially received considerable attention in the area of the SDSA. One can refer Burczynski [1], Lee [2], Choi et al. [3] and Li et al. [4] for a survey of research on the BEM-based SDSA. The methods can be classified into a semi-analytical and an analytical approach. In the semi-analytical approach, the state equations are first discretized into boundary elements and next the derivatives of the system matrices are derived analytically. In the analytical approach, the material derivatives [5] are taken to the state equations and sensitivity formulas are analytically derived on the continuum basis, next discretization followed only for the numerical implementation. The continuum approach can again be classified into the adjoint variable method (AVM) and the direct differentiation method (DDM). In the AVM, the shape sensitivity formulas are derived by introducing adjoint systems that are determined depending on the performance functional, whereas, in the DDM, the sensitivity of the state variables is directly calculated by solving the BIE for the design derivatives.

While most of the research on the SDSA has dealt with the elasticity problem, papers focusing on the SDSA of solids in thermal environment are not so many. Most of the published papers for the SDSA of thermal conduction problems belong to the analytical approach. Meric [6–8] has derived shape sensitivity formulas for thermoelastic and thermal conduction problems using the Lagrangian multiplier technique, the material derivative concept and the AVM. Park and Yoo [9] have applied the method of variational formulation [5] to the thermal conduction problem and derived a sensitivity formula, transforming the variational equation to an equivalent BIE. The SDSA based on the BIE formulation has been presented for thermoelasticity problem (Lee and Kwak [10,11]) and for thermal conduction problem (Lee et al. [12,13] and Lee [14]), using the AVM and the DDM. Aithal and Saigal [15] have presented the SDSA of Dirac delta type performance functional for the thermal conduction problem using the AVM. Sluzalec and Kleiber [16] have presented a shape sensitivity formula for a nonlinear thermal conduction problem with temperature-dependent thermal conductivity using the AVM. Kane and Wang [17,18] presented the semi-analytical approach for nonlinear thermal conduction problems with nonlinear boundary conditions and temperature-dependent thermal conductivity.

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Most of the previous BEM-based researches for the SDSA of thermal conducting solids are concerned with homogeneous problems. Present paper deals with the SDSA of inhomogeneous thermal conduction problems with zoned-interfaces of different materials. A cooling fin, a molding die, a composite tube and so on, consisting of several materials with different thermal conductivities can be examples of such problems. Choi et al. [3] presented the SDSA of an interface problem as applied to implant design in dentistry. Their method uses the BIE formulation and is confined to the elasticity problem. Dems and Mroz [19] have presented the SDSA of the two-dimensional thermal conduction problems with zoned-interfaces, and they derived a general shape sensitivity formula for varying interfaces using the variational formulation based on the finite element method. Recently, Gao and He [20] presented a finite-difference approach called as the complex-variable-differentiation method for the shape sensitivity of multi-region heat conduction using the BEM. The method of the SDSA in present paper is basically different from Dems and Mroz [19] and Gao and He [20] in that it is an analytic approach based on the BIE formulation. The material derivative concept is used to represent the shape variation of the external and interface boundaries. The shape sensitivity formula is derived using the boundary integral identity [12,13] and employing the AVM. The theoretical formulation is validated with an analytical example of a composite wall problem. Shape sensitivity is numerically analyzed and its accuracy is verified for three numerical examples: a compound cylinder, a thermal diffuser and a cooling fin problem. As application to numerical optimization, the sensitivity formula is incorporated to a nonlinear programming algorithm and optimum shapes are found for the thermal diffuser problem and the cooling fin problem.

2. Method of the SDSA

Consider a two-dimensional isotropic and inhomogeneous thermal conducting solid body $\Omega = \Omega^{(1)} \cup \Omega^{(2)}$ with zoned-interface boundary Γ_I, as shown in Fig. 1. The superscript (1) or (2) denotes a zone number identifying a material type. External boundary of Ω is represented as $\Gamma = \Gamma_E^{(1)} \cup \Gamma_E^{(2)}$, where $\Gamma_E^{(1)}$ and $\Gamma_E^{(2)}$ represents external boundary of each zone. The position in the solid body will be denoted by x when necessary. If the temperature is denoted by $T(x)$, then the heat flux vector representing the heat

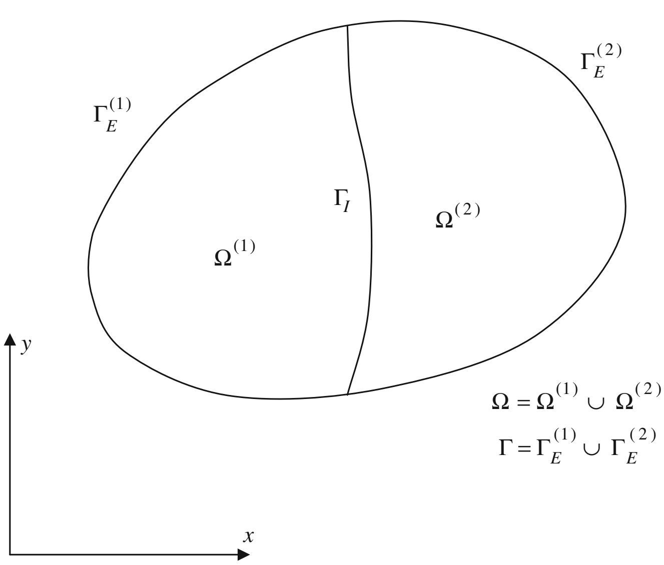

Fig. 1. A two-dimensional thermal conducting solid body with zoned-interface.

flow rate per unit area is expressed as

$$q_i = -kT_{,i} \tag{1}$$

where k is the thermal conductivity of the material. The indices in the tensor notations have a range of 1 and 2. For a case without heat source, the steady-state thermal equilibrium equation is given by

$$-q_{i,i} = 0 \text{ in } \Omega \tag{2}$$

A general thermal boundary condition is applied on the external boundary $\Gamma = \Gamma_T \cup \Gamma_q \cup \Gamma_h$ as

$$\begin{aligned} &T = T_0 \text{ at } \Gamma_T \\ &q \equiv q_i n_i = q_0 \text{ at } \Gamma_q \\ &q \equiv q_i n_i = h(T - T_\infty) \text{ at } \Gamma_h \end{aligned} \tag{3}$$

where q is the normal heat flux, n_i the outward unit normal vector on the boundary. Temperature T_0 is prescribed at Γ_T, normal heat flux q_0 at Γ_q and constant heat transfer coefficient h and surrounding temperature T_∞ at Γ_h. At the interface Γ_I, continuity of the temperature and the normal heat flux should be imposed as

$$\left.\begin{aligned} &T^{(1)} = T^{(2)} = T^c \\ &q^{(1)} = -q^{(2)} = q^c \end{aligned}\right\} \text{ at } \Gamma_I \tag{4}$$

Introducing an arbitrary system of temperature T^* and normal heat flux q^* and using the procedure in Lee et al. [12,13], one can derive the boundary integral identity (BII) applicable to the zoned-inhomogeneous thermal conduction problem as

$$\begin{aligned} &\int_{\Gamma_E^{(1)} \cup \Gamma_I} (T^{(1)} q^{(1)*} - q^{(1)} T^{(1)*}) ds = 0 \\ &\int_{\Gamma_E^{(2)} \cup \Gamma_I} (T^{(2)} q^{(2)*} - q^{(2)} T^{(2)*}) ds = 0 \end{aligned} \tag{5}$$

where ds represents integration with respect to x on the boundary. Above equations correspond to Green's second identity that holds between two thermal equilibrium states: one with $T^{(i)}$ and $q^{(i)}$, and the other with $T^{(i)*}$ and $q^{(i)*}$.

Now, consider a general performance functional arising in the shape optimization in the following form

$$\Phi = \int_\Gamma \psi(T,q) ds \tag{6}$$

We define the performance functional Φ on the external boundary Γ, taking into consideration that most of the shape optimization problems for the thermal conducting solids are concerned with the state variables on the external boundary. Shape variation in the shape optimization problem is defined as a change of the shape of the external boundary Γ and the interface boundary Γ_I. Using the material derivative concept [5], we can define the shape variation using a velocity vector $\mathbf{V} \equiv \{V_1, V_2\}$ and a time-like parameter τ as shown in Fig. 2. Taking the material derivative to Eq. (6), variation of Φ can be expressed as

$$\begin{aligned} \delta\Phi &= \int_\Gamma \left(\psi_T \dot{T} + \psi_q \dot{q}\right) ds + \int_\Gamma \psi V_{j,s} s_j ds \\ &= \int_{\Gamma_T} \psi_q \dot{q} ds + \int_{\Gamma_q} \psi_T \dot{T} ds + \int_{\Gamma_h} (\psi_T + h\psi_q) \dot{T} ds + \int_\Gamma \psi V_{j,s} s_j ds \end{aligned} \tag{7}$$

where ψ_T and ψ_q denote partial derivatives, and $\dot{T}$ and $\dot{q}$the material derivatives, respectively. V_j represents the design velocity vector, s_j a unit tangential vector on the boundary, and a subscript (,s) the differentiation in the tangential direction on the boundary. On the other hand, taking the material derivative to each BII in Eq. (5), summing the resulting two equations, and