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Uncertainty and sensitivity analysis of creep models for uncorrelated and correlated input parameters

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ABSTRACT

The intention of this paper is to evaluate the uncertainties and sensitivities of creep prediction models of standard concrete. The development of creep prediction models has been a field of extensive research and many different models have already been proposed. The four major models are: model GL2000 by Gardner and Lockman, model MC90 according to CEB-FIP Model Code 1990, model ACI209 according to the American Concrete Institute and model B3 by Bažant and Bajewa. First, a sensitivity study is performed in order to determine the parameters which mostly contribute to the uncertainties of the model prediction. This is done for uncorrelated and correlated input parameters and the differences are pointed out. Due to high parameter correlation, most standard sensitivity methods are not applicable and, therefore, a new method developed by Xu and Gertner is applied. Second, the uncertainties of the creep prediction for all of the models are compared and reveal significant differences. Due to the consideration of parameter and model uncertainties, a measure for the total variation of the model response is achieved. A special FEM code is developed to include the existing creep models in structural analysis. Utilising the FEM application, arbitrarily distributed creep strains, non-linear creep and single reinforcement bars can be taken into account. Finally, this FEM code is used and the prescribed creep models and uncertainty method are applied to a pre-stressed concrete bridge. The uncertainties of the loss of pretensioning force and axial shortening are calculated and show a reduction in comparison to uncertainties of pure creep strain. Together, the presented method proves its ability to determine uncertainties and sensitivities of models of time-dependent behaviour for uncorrelated and correlated parameters in an efficient way.

1. Introduction

The prediction of long-term deformation of concrete and reinforced concrete structures has been a field of extensive research for many decades and several different creep models have been developed so far. These models vary in terms of theory, complexity and described phenomena. Advanced creep formulations take into account non-linear creep, aging of concrete, additional damage due to creep, and secondary and tertiary creep phases. Moreover, they can simulate loading–unloading processes. A general approach to evaluate the quality of these different models for their specific design purposes does not yet exist and is the object of the authors' work. One part of this evaluation method is the estimation of sensitivities of the model prediction towards the input parameters and the calculation of uncertainties of the model's prognosis, presented in this paper.

The investigation of uncertainties concerning time-dependent behaviour of concrete has been the subject of much research. Madsen and Bažant [1] studied the parameter- and model uncertainties of creep model BP by Bažant, Kim and Panula. Diamantidis et al. [2] extended it to determine the influence of varying humidity on the creep coefficient of model C78, according to the Model Code 1978. Further uncertainty surveys of creep models were proposed by Bažant and Liu [3], using the effective Latin Hypercube Sampling. In [4], Bažant and Bajewa propose a simple approach to consider the creep uncertainties in the design of structures. Besides the material uncertainties, further external uncertainties were considered in the context of time-dependent deformation of reinforced concrete structures in [5–7]. Yang [8–10] determined the uncertainties and sensitivities of creep and shrinkage models ACI209 and MC90 and their effect on pre-stressed elements. Howells et al. [11] did an intensive study of sensitivities of creep and shrinkage models using local sensitivity measures. All the mentioned research work on uncertainties and sensitivities of creep of concrete has assumed uncorrelated input parameters and, neglected appropriate parameter correlation.

When assuming uncorrelated input parameters, there are many different techniques to determine global sensitivities. A good

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overview of global sensitivity analysis is given by Saltelli et al. [12,13]. Most common are the Response Surface Methods (RSM), variance-based methods considering first order effects (VBFO) and total effects (VBTE), Fourier Amplitude Sensitivity Test (FAST), and the Random Balance Design (RBD). The application of sensitivity methods to models including correlated input parameters is still studied today. The most strategies can be found in [14–18]. In 2008 Xu and Gertner [18] published an approach using linear regression models (RSM) to determine uncorrelated and correlated sensitivities. This is used in the following.

As mentioned before, all previous sensitivity and uncertainty analyses of creep models assumed uncorrelated input parameters. In the scope of this paper, the effect of the consideration of parameter correlation in the context of a global sensitivity and uncertainty analysis is presented, applying the method by Xu and Gertner. In the following, four different creep models are compared: model GL2000 by Gardner and Lockman [19], model according to ACI209 [20], model according to CEB-FIP Model Code 90 [21] and model B3 by Bažant and Bajewa [22]. All of the models are summarised in [23].

In Section 2 the creep models and their theory are briefly explained. Section 3 reveals the method of uncertainty modelling and the main parts of the sensitivity method by Xu and Gertner. The results of the sensitivities and uncertainties regarding the prediction of creep strains are presented in Section 4. A comparison of the models and the effect of parameter correlation are given. The effects of these uncertainties on the structural response of a pre-stressed concrete bridge is demonstrated in the last section, showing the uncertainties of the pretensioning force and the axial shortening caused by creep.

2. Creep models

The applied models are briefly explained in this section. The following equations are only valid for the determination of the creep strain ϵ_{cr} for constant stresses and have to be combined with the Boltzmann principle of superposition for varying stresses.

Model ACI209 assumes an ultimate creep value $\varphi_{\tau,\infty}$ and combines it with a hyperbolic time-function. The values of $d = 10$ for the addend and $\Psi = 0.6$ for the exponent are recommended, but can be modified in the ranges of $6 \leq d \leq 30$ and $0.4 \leq \Psi \leq 0.8$. $\varphi_{\tau,\infty}$ is defined by the corrections factor γ_c, depending on loading age τ, humidity RH, concrete composition, geometry, and fresh and hardened concrete properties. The creep value here refers to Young's modulus at the concrete age at loading E_τ

$$\epsilon_{cr}(t,\tau,\sigma) = \frac{\varphi_\tau(t,\tau)}{E_\tau}\sigma = \frac{\frac{(t-\tau)^\Psi}{d+(t-\tau)^\Psi}\varphi_{\tau,\infty}(\tau)}{E_\tau}\sigma, \tag{1}$$

with

$$\varphi_{\tau,\infty} = 2.35\,\gamma_c = 2.35\,\gamma_\tau\gamma_{RH}\gamma_{v-r}\gamma_{v/s}\gamma_{sl}\gamma_{fa-r}. \tag{2}$$

Model MC90 is quite similar to ACI209. The differences are the ultimate creep value $\varphi_{28,\infty}$, referring to E_{28}, and a fixed hyperbolic time-function with the exponent 0.3. Furthermore, an over-proportionality factor $F_\Omega(\sigma)$ for stress levels exceeding $0.4f_c$ is integrated in $\varphi_{28,\infty}$. The creep coefficient depends on the concrete strength, the humidity and the type of cement,

$$\epsilon_{cr}(t,\tau,\sigma) = \frac{\varphi_{28}(t,\tau,\sigma)}{E_{28}}\sigma, \tag{3}$$

with

$$\varphi_{28}(t,\tau,\sigma) = F(\sigma)\varphi_{RH}\beta_{f_{cm}}\beta_{\tau_{eff}}\left[\frac{t-\tau}{\beta_H+(t-\tau)}\right]^{0.3}. \tag{4}$$

Model GL2000 describes the increase of creep strains by the creep coefficient φ_{28}, similar to Eq. (3). φ_{28} is defined only by humidity, cement type and a geometrical factor. In contrast to the first models, no ultimate creep value is assumed, but rather a continuous increase of creep strains over time. The time progress is defined by hyperbolic and hyperbolic-exponential functions. $\phi(t_c)$ considers drying before loading

$$\begin{aligned}\varphi_{28}(t,\tau) &= \phi(t_c)\left[2\left(\frac{(t-\tau)^{0.3}}{(t-\tau)^{0.3}+14}\right)+\left(\frac{7}{\tau}\right)^{0.5}\left(\frac{t-\tau}{t-\tau+7}\right)^{0.5}\right]\\ &\quad+\phi(t_c)\left[2.5\left(1-1.086RH^2\right)\left(\frac{t-\tau}{t-\tau+0.15\left(\frac{V}{S}\right)^2}\right)^{0.5}\right].\end{aligned} \tag{5}$$

Model B3 is the model with the highest physical background, based upon the solidification theory by Bažant and Prasannen [24,25]. It explicitly distinguishes creep into basic and drying creep. Basic creep is defined by aging and non-aging visco-elastic and visco-plastic (flow) compliance. The material parameters $q_1 \cdots q_5$ depend on the concrete composition and the concrete strength. The total strains are defined as

$$\epsilon(t,\tau,\sigma) = [q_1 + F_\Omega(\sigma)C_0(t,\tau) + F_\Omega(\sigma)C_d(t,\tau,t_d)]\,\sigma, \tag{6}$$

where q_1 is the instantaneous compliance, C_0 is the basic creep compliance, described by q_2, q_3, and q_4, and C_d is the drying creep compliance, defined by q_5. Over-proportionality for higher stress levels is taken into account by $F_\Omega(\Omega)$. In contrast to the other three models, the time-independent compliance is defined by $0.6/E_\tau$ instead of $1/E_\tau$. This effectively separates the short-term creep strains, included in the usual experimental test to determine Young's modulus.

3. Strategy of sensitivity and uncertainty analysis

Sensitivity- and uncertainty analyses are widely-used methods to investigate the reliability of a model's prognosis. Saltelli et al. [12,26] applies an uncertainty analysis in the study to determine how much uncertainty is in the model output (uncertainty analysis) and where it comes from (sensitivity analysis). This meaning will be used in the following.

3.1. Uncertainty analysis

In this paper the uncertainties of the model prognosis are distinguished between parameter- and model uncertainties. Due to stochastic input parameters, e.g. the variation of concrete strength, the model output includes uncertainties. Furthermore, assuming deterministic input parameters, model prognoses show deviation from experimental results. This is a result of the inaccuracy, or the error of the model. By comparing experimental tests to the model prediction, we can derive a general model uncertainty, representing the general variation of the model prognosis to the experimental creep tests. According to Madsen and Bažant [1], the total uncertainty observed from comparison of calculations to experiments Ψ_{cr} can be divided into uncertainties of measurements Ψ_β, internal uncertainty of the creep phenomenon Ψ_α as well as the uncertainty of the model prognosis Ψ^*_{cr}

$$\Psi_{cr} = \Psi^*_{cr}\Psi_\alpha\Psi_\beta. \tag{7}$$

Consequently, the uncertainty of the model prognosis can be derived by decomposing the variances

$$CV^2_{\Psi,cr} = CV^{2*}_{\Psi,cr} + CV^2_{\Psi,\alpha} + CV^2_{\Psi,\beta}. \tag{8}$$

The variation from experiments $CV_{\Psi,cr}$ are determined by Gardner [27] for the investigated creep models through a comparison