Electrical Power and Energy Systems 32 (2010) 969–974

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Studies of cases in power systems by Sensitivity Analysis oriented by OPF

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ARTICLE INFO

Article history:
Received 23 January 2008
Received in revised form 7 November 2009
Accepted 23 February 2010

Keywords:
Sensitivity Analysis
Optimal Power Flow
Power system
Newton's method

ABSTRACT

This paper presents studies of cases in power systems by Sensitivity Analysis (SA) oriented by Optimal Power Flow (OPF) problems in different operation scenarios. The studies of cases start from a known optimal solution obtained by OPF. This optimal solution is called base case, and from this solution new operation points may be evaluated by SA when perturbations occur in the system. The SA is based on Fiacco's Theorem and has the advantage of not be an iterative process. In order to show the good performance of the proposed technique tests were carried out on the IEEE 14, 118 and 300 buses systems.

1. Introduction

One of the main tasks of a power system operator is to manage the system in such a way that its operation be safe and reliable. The availability of tools that aid the operator in its task of maintaining the system operating safely is of fundamental importance for utilities. Studies of the present and future system conditions should be carried out and, furthermore, in many cases they should be done rapidly and precisely in order that decisions may be taken. In this context, Sensitivity Analysis (SA) emerges as an efficient instrument for studies of the planning of power electrical system operations given that it demonstrates the future performance of the system in different situations.

In specialized literature, there are very few proposals about the application of SA in power systems. One of the first papers on this line of research is the one proposed by Peschon et al. [1], in which studies of sensitivity between the magnitude of voltage and the injection of reactive in certain buses of the system were carried out. The Jacobian matrix, calculated in the system operation point, provided the sensitivity relations that were of interest to the study. Kishore and Hill [2], using the sensitivity matrix showed in [1], elaborated a technique for optimal reactive allocation in power systems. Wojciechowski [3] applied the Tellegen Theorem aiming at developing an efficient SA method. Gribik et al. [4] and Belati et al. [5] made a study of SA oriented by Optimal Power Flow (OPF) problems. The OPF problem widely divulged in literature [6–10] has the objective of minimizing a function and, at the same time, of satisfying a set of physical and operational constraints in power systems. As a solution, it provides the optimal operation point for the electrical network for a given generation and load configuration of the system. If perturbations occur in the electrical system, such as a variation in demand or alterations in operational limits, a new operation point should be obtained. This new point can be founded by running again the OPF program or by a SA technique applied to the OPF. SA oriented by OPF requires a known optimal point, which is obtained from the OPF program. The solution provided by the OPF is called base case solution and after perturbations occur new optimal points can be obtained via SA. The main advantage of using the SA approach is that, unlike the OPF, it is not iterative and it does not demand to estimate the initial barrier parameter and its correction factor. Thus, it allows not only to follow the load variations, but also to analyze, fast and efficiently, different strategies of adjustments.

In this paper we presented the SA oriented by OPF to accomplish studies of cases in power system for different operation scenarios. The OPF was solved by primal–dual logarithmic barrier method [11,12] and the SA formulation is based on a theorem proposed by Fiacco [13]. The developed models of both optimization problems – OPF and SA – were the same. The minimized objective function was active power losses in the transmission system. The set of constraints was given by: balance equations and operational limits of magnitude voltage, LTC' taps, active and reactive power generations and active power flow in transmission lines. Aiming at avoiding the complexity of the problem, transmission line constraints are usually avoided in studies [4], but we chose to implement this constraint in order to verify the efficiency of SA when the model of the problem contains a greater number of constraints. We

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doi:10.1016/j.ijepes.2010.02.008

Nomenclature

$\boldsymbol{x}$	control and state variable	L	Lagrangian function
f	objective function	$\boldsymbol{\lambda}$, $\boldsymbol{\pi}$	Lagrange multiplier vectors
$\boldsymbol{g}$	equality constraints	α_p	primal step length
$\boldsymbol{h}$	inequality constraints	α_d	dual step length
$\boldsymbol{s}$	slack vector	ε_1	scalar vector of the equality perturbations
μ	barrier parameter	ε_2	scalar vector of the inequality perturbations
β	correction factor		

also point out in this work the complete implementation of Fiacco's Theorem, with the possibility of perturbing equality and inequality constraints. Studies of cases were carried out in the IEEE 14, 118 and 300 buses systems, where the efficiency of the presented technique was verified. This paper is organized as follows: the OPF problem and a brief review of the primal–dual logarithmic barrier method are presented, next the formulation used for the SA is shown and, finally, the studies of cases and the conclusions are presented.

2. Optimal Power Flow formulation

The OPF can be presented as:

$$\begin{aligned} &\text{Minimize } f(\boldsymbol{x}) \\ &\text{subject to } \boldsymbol{g}(\boldsymbol{x}) = 0 \\ &\boldsymbol{h}(\boldsymbol{x}) \leqslant 0 \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in R^n$ is the control and state variable vector, i.e., voltage magnitude, phase angles and LTC's taps. The scalar $f(\boldsymbol{x})$ is the objective function and represents the active power losses in transmission system. The vector function $\boldsymbol{g}(\boldsymbol{x}) \in R^m$, where $m < n$, is the power flow equations. The vector function $\boldsymbol{h}(\boldsymbol{x}) \in \mathrm{R}^p$ is the operational limits of voltage magnitude, LTC's taps, active and reactive power generation and active and reactive power flows in transmission lines.

The OPF is a very large and very difficult mathematical programming problem. Almost every mathematical programming approach that can be applied to this problem has been attempted. The primal–dual logarithmic barrier method is one of these approaches. The next subsection shows a brief review of this method.

2.1. Primal–dual logarithmic barrier method

To solve problem (1) by the primal–dual logarithmic barrier method, it is necessary to transform all inequality constraints into equality constraints by adding non-negative slack vectors $\boldsymbol{s} \geqslant 0$, $\boldsymbol{s} \in \mathrm{R}^p$, so the problem can be written:

$$\begin{aligned} &\text{Minimize } f(\boldsymbol{x}) \\ &\text{subject to } \boldsymbol{g}(\boldsymbol{x}) = 0 \\ &\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s} = 0 \\ &\boldsymbol{s} \geqslant 0 \end{aligned} \tag{2}$$

The non-negativity conditions $\boldsymbol{s} \geqslant 0$ as (2) are handled by incorporating them into logarithmic barrier terms, given:

$$\begin{aligned} &\text{Minimize } f(\boldsymbol{x}) - \mu \sum_{j=1}^{p} \ln(\boldsymbol{s}_j) \\ &\text{subject to } \boldsymbol{g}(\boldsymbol{x}) = 0 \\ &\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s} = 0 \end{aligned} \tag{3}$$

where $\boldsymbol{s} > 0$ and $\mu > 0$ is the barrier parameter that is monotonically decreased to zero as iterations progress, i.e., $\mu_0 > \mu_1 > \ldots > \mu_\infty \to 0$. The sequence of parameters generates a sequence of sub-problems given by (3) and, under regularity assumptions, as $\mu \to 0$ the sequence of solutions approaches the local optimal solution.

The Lagrangian, L, associated with the problem is given by:

$$L = f(\boldsymbol{x}) - \mu \sum_{j=1}^{p} \ln(\boldsymbol{s}_j) + \boldsymbol{\lambda}^T \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{\pi}(\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}) \tag{4}$$

where $\boldsymbol{\lambda} \in \mathrm{R}^m$ and $\boldsymbol{\pi} \in R^p$ are the vectors of Lagrange multipliers, called dual variables. The first-order necessary conditions are applied to the Lagrangian function (4), generating nonlinear system equations:

$$\nabla_z L = 0 \tag{5}$$

where $\boldsymbol{z} = [\boldsymbol{x}, \boldsymbol{s}, \boldsymbol{\lambda}, \boldsymbol{\pi}]$

Eq. (5) is a set of nonlinear equations that can be solved using Newton's method to obtain the correction vectors $\Delta\boldsymbol{z}$. The correction vectors are used to update $\boldsymbol{x}$, $\boldsymbol{s}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\pi}$, as follows:

$$\begin{aligned} \boldsymbol{x}^{k+1} &= \boldsymbol{x}^k +, \alpha_p \Delta\boldsymbol{x}^k \\ \boldsymbol{s}^{k+1} &= \boldsymbol{s}^k + \alpha_p \Delta\boldsymbol{s}^k \\ \boldsymbol{\lambda}^{k+1} &= \boldsymbol{\lambda}^k + \alpha_d \Delta\boldsymbol{\lambda}^k \\ \boldsymbol{\pi}^{k+1} &= \boldsymbol{\pi}^k + \alpha_d \Delta\boldsymbol{\pi}^k \end{aligned} \tag{6}$$

where the step length $\alpha \in (0, 1)$ is chosen as follows:

$$\alpha_p^{\max} = \min\left\{\frac{\boldsymbol{s}}{|\Delta\boldsymbol{s}|} : \Delta\boldsymbol{s} < 0\right\} \tag{7}$$

$$\alpha_d^{\max} = \min\left\{\frac{\boldsymbol{\pi}}{|\Delta\boldsymbol{\pi}|} : \Delta\boldsymbol{\pi} < 0\right\} \tag{8}$$

$$\alpha = \min\{\tau\alpha_p^{\max}\tau\alpha_d^{\max}, 1.0\} \tag{9}$$

where the scalar $\tau \in (0, 1)$ is a safety factor to ensure that the next point will satisfy the feasibility conditions. $\tau = 0.9995$ is an empirical value which, according to [14], can be derived from the expression $1 - 1/(9\sqrt{\mathrm{nc}})$, where nc is the total number of inequality constraints in the problem.

A critical point in the primal–dual algorithm is the choice of the barrier parameter μ. The condition $\nabla_z L = 0$ suggests that μ could be reduced on the basis of a predicted decrease of the complementarily gap.

A strictly feasible starting point is not mandatory, but the conditions $\boldsymbol{s} > 0$ and $\boldsymbol{\pi} > \mathbf{0}$ must be satisfied at every point. The process of optimization is finished when KKT conditions are satisfied.

3. Sensitivity Analysis

The SA formulation is based on a theorem proposed by Fiacco [13]. In this work, we consider the complete implementation of Fiacco's Theorem with the possibility of perturbing equality and inequality constraints of the OPF problem (1). The perturbation ε_1, that is, power increments on the buses, is associated with the equality constraints, $\boldsymbol{g}(\boldsymbol{x})$. The perturbation ε_2 is associated with the inequality constraints, $\boldsymbol{h}(\boldsymbol{x})$, that is, the binding limit of active