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Sensitivity analysis applied to slope stabilization at failure

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ABSTRACT

This article discusses how sensitivity analysis is a sound assessment tool for selecting the most efficient stabilization method of slopes at failure. A discretized form of the variational approach is used not only for performing sensitivity analysis but to locate the critical slip surface, i.e., the sensitivity analysis is carried out in the same way as it is done in optimization problems. This method supplies a robust formulation and methodology for obtaining the sensitivities of the safety factor with respect to both the soil parameters and the slope profile, stating the slope stabilization design as a relatively simple minimization problem. Two well known examples, as the Selset landslide and the Sudbury Hill slip are used to illustrate the application of the method and to highlight both its capabilities and limitations.

1. Introduction

Today, engineers are not completely satisfied with the solutions to given problems and they also require knowledge of how these solutions depend on data. Thus, for instance, in minimization problems, such as those associated with slope stability, it is not enough to know the optimal value of the objective function (the safety factor) and the solution (slip line) where the minimum is attained. In this sense, approaches as the "safety maps" introduced by Baker and Leshchinsky [1] provides a valuable information to determine how and how much specific changes in the parameters of the system modify both the optimal objective function value and the optimal solution.

In the field of slope stability, sensitivity analysis is generally conducted by means of a series of calculations in which each significant parameter is varied systematically over its maximum credible range in order to determine its influence upon the safety factor [2]. If one is interested in characterizing the variation in safety when encounter minor modifications in the parameters, these incremental techniques define an approximation of the safety factor gradient. In this case, for discrete problems (i.e., slices) the sensitivity may be calculated in a simpler and compact way by using the techniques that have been developed in the area of non-linear optimization [3]. When dealing with continuous problems as those linked to the variational approach of slope stability analysis, the formulation put forth by Castillo et al. [4] can be used.

In any event, regardless of how the sensitivity analysis is done, when instability occurs, a sensitivity analysis allows to know which qualitative or quantitative actions are more appropriate to stabilize the given slope. Therefore, the sensitivity analysis is a useful tool able to provide a sound assessment for the selection of the slope stabilization method. Our main objective in this article is to analyze the use of this sensitivity analysis tool.

2. Conceptual basis of the method

Both when limit equilibrium methods are used, and when the kinematic approach of limit analysis is applied, if the safety factor is defined as the ratio of the shear stress of the soil to the shear stress at failure, slope stability is generally evaluated as a ratio:

$$F = \frac{S}{D} = \frac{\int_a^b G(x, y(x), y'(x), y_G(x), y'_G(x), u(x, y); \boldsymbol{p})\, dx}{\int_a^b Q(x, y(x), y'(x), y_G(x), y'_G(x), u(x, y); \boldsymbol{p})\, dx} \quad (1)$$

For the two-dimensional collapse mechanism defined in Fig. 1, a and b are the x-coordinates of the sliding line end points, $y_G(x)$ is the ground profile (ordinate at point x), $y(x)$ is the ordinate of the sliding line at point x, and $y'(x)$ is its first derivative. The $u(x, y)$ function defines the distribution of the soil water pressure. Finally, vector $\boldsymbol{p}$ groups all the parameters together. In principle, it could be a vectorial field (if, for example, the spatial variation of the strength parameters is taken into account), though what usually happens is that it contains only a discrete number of parameters which are constant throughout the entire domain. G and Q are two functionals that define the actions over the system. When limit equilibrium methods are used, actions are identified with forces

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Nomenclature

a, b	x-coordinates of the sliding line end points	S	stabilizing actions
a_W, b_W	x-coordinates of the water table exit point at the slope side, and at the slope top	$\boldsymbol{s}$	vector of sensitivities
C	associated cost	t	length of the interface between blocks
c	cohesion	u	pore water pressure at the base of the blocks
D	driving actions	v	virtual displacement
E	horizontal stabilization force	x	abscissa coordinate
F	safety factor	y	sliding line
F^+	value of F when the ith component of $\boldsymbol{p}$ is increased by δp_i	y'	first derivative of the sliding line
F^-	value of F when the ith component of $\boldsymbol{p}$ is decreased by δp_i	y_G	ground profile
F^*	value of F for slopes at collapse	$y_G{}'$	first derivative of the ground profile
G	function that defines the stabilizing actions	α	angle of inclination of the block base with respect to the horizontal
H	slope height	Δx_k	horizontal width of blocks
l	length of the block base	δF	increase of safety factor
n	number of blocks	δF^*	increase of safety factor for slopes at collapse
n_p	number of components of vector $\boldsymbol{p}$	$\delta\boldsymbol{p}$	perturbation in the value of the parameters
$\boldsymbol{p}$	vector of parameters	ε	compact notation of the differential operator defined in Eq. (11)
Q	function that defines the driving actions	ϕ	angle of internal friction
		θ	slope angle

or moments. On the other hand, if the kinematic approach of limit analysis is used, the actions may be identified with the internal dissipation of energy, and the external work. This procedure is referred to as the "kinematical approach" in this paper.

The slope stability problem can be stated as the minimization of the "safety functional" (as Baker and Garber [5], termed the quotient F of Eq. (1)) to find the safety factor:

$$F = Min_{y(x),\sigma(x)} \quad \{F[y(x),\sigma(x)]\} \tag{2}$$

where $\sigma(x)$ represents the distribution of stresses along $y(x)$. To minimize the quotient functional, the Petrov method can be used. Petrov [6] showed that stationary "points" (functions), of a ratio can be obtained by extremizing an auxiliary functional $R = R(F_S) \equiv G - F_S\,Q$, where F_S is the unknown minimum value of the ratio G/Q, which can be evaluated from the constraint $R(F_S) = 0$. The minimization can also be done by an iterative method. This is the case of Baker [7], when he applies the dynamic programming algorithm iteratively, i.e., assuming a value for Fs, and establishes the critical slip surface $y_{CR}\,(x|F_S)$, by minimizing $R[y(x)|F_S]$, obtaining a new estimate of F_S by applying the Spencer's procedure to this critical slip surface, and repeating the process until the assumed and resulting values of F_S are equal. Although the two preceding methods have proved its efficiency, for the approach proposed in this paper is of interest to minimize the quotient functional directly:

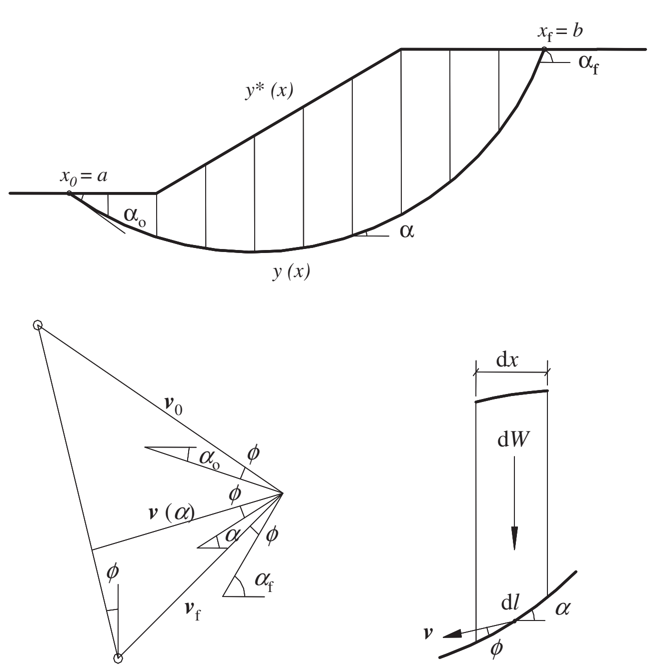

Fig. 1. Collapse mechanism, typical slice, and hodograph of the movement.

$$\frac{\partial F}{\partial p_i} = \frac{\partial (S/D)}{\partial p_i} = \frac{\frac{\partial S}{\partial p_i} D - S\frac{\partial D}{\partial p_i}}{D^2} = \frac{1}{D}\left(\frac{\partial S}{\partial p_i} - F_S\frac{\partial D}{\partial p_i}\right) \tag{3}$$

Then, standard optimization packages, as GAMS [8] for example, can be used. This allows to convert F_S into one more variable that has to satisfy its definition (Eq. (1)). Therefore, it is not needed to worry about the method of solution, because the method implemented in the software package takes this into account as one constraint. As a result, in keeping with the proposal of Castillo et al. [9], the components of the vector of sensitivities $\boldsymbol{s}$ of the objective function with respect to $\boldsymbol{p}$ can be defined as:

$$s_i = \frac{\partial F}{\partial p_i} = \frac{\frac{1}{D}\int_a^b \left(\frac{\partial G}{\partial p_i} - F\frac{\partial Q}{\partial p_i}\right) dx}{1 - \frac{1}{D}\int_a^b \left(\frac{\partial G}{\partial F} - F\frac{\partial Q}{\partial F}\right) dx} \tag{4}$$

where for simplicity, the arguments of the functionals have been omitted. The safety factor local sensitivities are defined as the partial derivatives of the safety factor with respect to the parameter being studied. The partial derivatives are calculated at the optimum value. Thus, these sensitivities provide only a linear approximation in a neighborhood of the optimal point, and they only indicate the direction of the action to be taken. Since small property increments δp_i would produce significant changes in the critical slip surface, the simultaneous variation of all the variables and functions involved is taken into account, included the slip surface.

When idealized examples are analyzed, the slope stability can be studied analytically, and the sensitivity of the solution can be characterized on the basis of the parameters using Eq. (4). In many practical applications, however, these analytical computations cannot be carried out. In these cases, the S/D ratio is generally discretized by means of slices (limit equilibrium method) or blocks (kinematical approach). When these approaches are adopted, the