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Sensitivity Analysis for Structures Subjected to Stationary Random Excitations

Y. Zhao[1a], F. Y. Wang[1b]

[1] *Department of Engineering Mechanics, State Key Laboratory of Industrial Equipment and Structural Analysis, Dalian University of Technology*

Abstract

A method for sensitivity analysis of stationary random vibration is proposed within the framework of pseudo-excitation method (PEM). As the stationary random vibration analysis is transformed into harmonic vibration analysis，therefore the sensitivity analysis of stationary random responses is transformed into sensitivity analysis of pseudo harmonic responses of structures subjected to pseudo harmonic excitations. Using the PEM principle once again, the formulae for calculating the sensitivities of power spectrum density (PSD) and variances of random responses are derived. Based on the above work, the study of statistical characteristics of stationary random responses for structures with uncertain parameters is carried out.

Keywords: random vibration, pseudo-excitation method, sensitivity

1. Introduction

The random vibration analysis of structures with uncertain parameters subject to random excitations has received much attention in recent years, and various methods for solving this problem have been proposed. Generally speaking, these methods can be specified as Monte Carlo simulation (Chakraborty and Dey 1996 1998, Zhao and Chen 2000, Bhattacharyya and Chakraborty 2002), orthogonal expansion methods (Jensen and Iwan 1992, Papadimitriou and Katafygiotis 1995) and perturbation methods (Lin and Yi 2001, Hosseini and Khadem 2007, Śniady et al. 2008). Although Monte Carlo simulation can be used in such double stochastic problems, it requires a large number of samples to guarantee the accuracy of results, which leads to very low computational efficiency. In the orthogonal expansion method, the structural

[a] Corresponding author: Email: yzhao@dlut.edu.cn
[b] Email: fy11423@sina.com

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response is expanded in terms of a series of orthogonal polynomials to obtain statistics of the stochastic response. If the problem involves many stochastic parameters, or if some stochastic parameters have large coefficients of variation (COV), the orthogonal polynomial will include many terms so that the computational cost will increase remarkably. The perturbation method has advantages in practical applications because of its high efficiency and simplicity. The accuracy of sensitivity analysis for random responses is the decisive factor for the reliability of the perturbation method.

In this paper, the perturbation method is combined with PEM (Lin et al. 2001 2005, Zhao and Lin 2008) to study the statistical characteristics of responses for structures with uncertain parameters subject to stationary random excitations. Not only will the computational accuracy increase, but also the analysis scale can meet the requirements of practical engineering structures. The vibration analysis of stationary random excitations can be transformed by PEM to the analysis of harmonic excitations. Similarly, the double random problems can be transformed into single random problems. Accurate formulae for the sensitivities of structural stochastic responses are deduced within the framework of accurate and efficient PEM to give the perturbation expressions. Finally, the method is applied to a space truss under stationary random excitations to calculate the mean values and standard deviations of the variances of structural responses. The accuracy of the proposed method is illustrated by comparison with Monte Carlo simulation.

2. Pseudo Excitation Method for Stationary Random Responses with Uncertain Parameters

The equations of motion for linear structures with uncertain parameters subjected to stationary random excitations are

$$\boldsymbol{M}(\boldsymbol{\alpha})\ddot{\boldsymbol{y}}(\boldsymbol{\alpha},t)+\boldsymbol{C}(\boldsymbol{\alpha})\dot{\boldsymbol{y}}(\boldsymbol{\alpha},t)+\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{y}(\boldsymbol{\alpha},t)=\boldsymbol{F}(t) \tag{1}$$

Constituting the pseudo-excitation $\tilde{\boldsymbol{F}}(t)=\boldsymbol{F}_0\sqrt{S_{ff}(\omega)}e^{\mathrm{i}\omega t}$, and substituting it into Eq. (1) yields

$$\boldsymbol{M}(\boldsymbol{\alpha})\ddot{\tilde{\boldsymbol{y}}}(\boldsymbol{\alpha},\omega,t)+\boldsymbol{C}(\boldsymbol{\alpha})\dot{\tilde{\boldsymbol{y}}}(\boldsymbol{\alpha},\omega,t)+\boldsymbol{K}(\boldsymbol{\alpha})\tilde{\boldsymbol{y}}(\boldsymbol{\alpha},\omega,t)=\boldsymbol{F}_0\sqrt{S_{ff}(\omega)}e^{\mathrm{i}\omega t} \tag{2}$$

According to PEM, the PSD matrix of the random displacement vector $\boldsymbol{y}(\boldsymbol{\alpha},\omega)$ is (Lin et al. 2001 2005)

$$\boldsymbol{S}_{yy}(\boldsymbol{\alpha},\omega)=\tilde{\boldsymbol{y}}(\boldsymbol{\alpha},\omega)^{*}\,\tilde{\boldsymbol{y}}(\boldsymbol{\alpha},\omega)^{\mathrm{T}} \tag{3}$$

The variance of the response is

$$v(\boldsymbol{\alpha})=\sigma^{2}(\boldsymbol{\alpha})=2\int_0^{\infty}S_{yy}(\boldsymbol{\alpha},\omega)d\omega \tag{4}$$

3. Sensitivities for stationary random responses with respect to uncertain structural parameters

3.1. The Sensitivities for Pseudo-responses with Respect to Uncertain Parameters

If $\boldsymbol{\Phi}_{\boldsymbol{\alpha}}$ is the modal matrix consisting of the first q normalized modes, then setting $\tilde{\boldsymbol{y}}=\boldsymbol{\Phi}_{\boldsymbol{\alpha}}\tilde{\boldsymbol{u}}$ gives the modal equation of motion

$$\ddot{\tilde{\boldsymbol{u}}}(\boldsymbol{\alpha},\omega)+\hat{\boldsymbol{C}}(\boldsymbol{\alpha})\dot{\tilde{\boldsymbol{u}}}(\boldsymbol{\alpha},\omega)+\hat{\boldsymbol{K}}(\boldsymbol{\alpha})\tilde{\boldsymbol{u}}(\boldsymbol{\alpha},\omega)=\tilde{\boldsymbol{f}}(t) \tag{5}$$

The first order sensitivity of the $\tilde{\boldsymbol{y}}$ with respect to the uncertain parameter α_i is

$$\frac{\partial\tilde{\boldsymbol{y}}}{\partial\alpha_i}=\frac{\partial\tilde{\boldsymbol{y}}}{\partial\tilde{\boldsymbol{u}}}\frac{\partial\tilde{\boldsymbol{u}}}{\partial\alpha_i}=\boldsymbol{\Phi}_{\boldsymbol{\alpha}}\frac{\partial\tilde{\boldsymbol{u}}}{\partial\alpha_i} \tag{6}$$

where $\dfrac{\partial\tilde{\boldsymbol{u}}}{\partial\alpha_i}$ is the first order sensitivity of the modal pseudo-displacement.