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Sensitivity analysis for generalized nonlinear parametric (A, η, m)-maximal monotone operator inclusion systems with relaxed cocoercive type operators☆

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ABSTRACT

By using Lim's inequalities, Nadler's results, the new parametric resolvent operator technique associated with (A, η, m)-maximal monotone operators, in this paper, the existence theorem for a new class of generalized nonlinear parametric (A, η, m)-maximal monotone operator inclusion systems with relaxed cocoercive type operators in Hilbert spaces is analyzed and established. Our results generalize sensitivity analysis results of other recent works on strongly monotone quasi-variational inclusions, nonlinear implicit quasi-variational inclusions and nonlinear mixed quasi-variational inclusion systems in Hilbert spaces.

1. Introduction

For $i = 1, 2$, let $\mathbb{X}_i$ be real Hilbert space, Λ_i be nonempty open subset of $\mathbb{X}_i$ in which the parameter λ_i take values, and let $S, E : \mathbb{X}_1 \times \Lambda_1 \to 2^{\mathbb{X}_1}$ and $T, G : \mathbb{X}_2 \times \Lambda_2 \to 2^{\mathbb{X}_2}$ be multi-valued operators, and $g_i, p_i : \mathbb{X}_i \times \Lambda_i \to \mathbb{X}_i$, $N^1 : \mathbb{X}_1 \times \mathbb{X}_2 \times \Lambda_1 \to \mathbb{X}_1$ and $N^2 : \mathbb{X}_1 \times \mathbb{X}_2 \times \Lambda_2 \to \mathbb{X}_2$ be single-valued operators. Suppose that for $i = 1, 2$, $A_i : \mathbb{X}_i \to \mathbb{X}_i$, $\eta_i : \mathbb{X}_i \times \mathbb{X}_i \to \mathbb{X}_i$ and $M^i : \mathbb{X}_i \times \mathbb{X}_i \times \Lambda_i \to 2^{\mathbb{X}_i}$ are any nonlinear operators such that for all $(\varpi, \lambda_1) \in \mathbb{X}_1 \times \Lambda_1$, $M^1(\cdot, \varpi, \lambda_1) : \mathbb{X}_1 \to 2^{\mathbb{X}_1}$ is an (A_1, η_1, m_1)-maximal monotone operator with $(g_1 - p_1)_{\lambda_1}(\mathbb{X}_1) \cap dom(M^1(\cdot, \varpi, \lambda_1)) \neq \emptyset$ and for all $(w, \lambda_2) \in \mathbb{X}_2 \times \Lambda_2$, $M^2(\cdot, w, \lambda_2) : \mathbb{X}_2 \to 2^{\mathbb{X}_2}$ is an (A_2, η_2, m_2)-maximal monotone operator with $(g_2 - p_2)_{\lambda_2}(\mathbb{X}_2) \cap dom(M^2(\cdot, w, \lambda_2)) \neq \emptyset$, respectively, where $(g_i - p_i)_{\lambda_i}(v) = (g_i - p_i)(v(\lambda_i), \lambda_i)$ for $\lambda_i \in \Lambda_i$ and $v(\lambda_i) \in \mathbb{X}_i$. Throughout this paper, unless otherwise stated, we shall consider the following generalized nonlinear parametric (A, η, m)-maximal monotone operator inclusion systems:

For each fixed $\lambda_i \in \Lambda_i$ $(i = 1, 2)$, find $(u(\lambda_1), v(\lambda_2)) \in \mathbb{X}_1 \times \mathbb{X}_2$ such that $x(\lambda_1) \in S_{\lambda_1}(u)$, $y(\lambda_2) \in T_{\lambda_2}(v)$, $z \in E_{\lambda_1}(u)$, $\omega(\lambda_2) \in G_{\lambda_2}(v)$ and

$$\begin{cases} 0 \in N^1(u(\lambda_1), y(\lambda_2), \lambda_1) + M^1_{\lambda_1}((g_1 - p_1)_{\lambda_1}(u), z), \\ 0 \in N^2(x(\lambda_1), v(\lambda_2), \lambda_2) + M^2_{\lambda_2}((g_2 - p_2)_{\lambda_2}(v), \omega), \end{cases} \tag{1.1}$$

where $M^i_{\lambda_i}(u, v) = M^i(u(\lambda_i), v(\lambda_i), \lambda_i)$ for all $(u, v, \lambda_i) \in \mathbb{X}_i \times \mathbb{X}_i \times \Lambda_i$, $i = 1, 2$.

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Example 1.1. If $E : \mathbb{X}_1 \times \Lambda_1 \to \mathbb{X}_1$ and $G : \mathbb{X}_2 \times \Lambda_2 \to \mathbb{X}_2$ are single-valued operators, then for each fixed $\lambda_i \in \Lambda_i$ $(i = 1, 2)$, the problem (1.1) reduces to finding $(u(\lambda_1), v(\lambda_2)) \in \mathbb{X}_1 \times \mathbb{X}_2$ such that $x(\lambda_1) \in S_{\lambda_1}(u), y(\lambda_2) \in T_{\lambda_2}(v)$ and

$$\begin{cases} 0 \in N^1(u(\lambda_1), y(\lambda_2), \lambda_1) + M^1_{\lambda_1}((g_1 - p_1)_{\lambda_1}(u), E_{\lambda_1}(u)), \\ 0 \in N^2(x(\lambda_1), v(\lambda_2), \lambda_2) + M^2_{\lambda_2}((g_2 - p_2)_{\lambda_2}(v), G_{\lambda_2}(v)). \end{cases} \tag{1.2}$$

The problem (1.2) is called a system of general nonlinear parametric mixed quasi-variational-like inclusions, which was studied by us [1] when $g_i - p_i = E = G \equiv I$ for $i = 1, 2$, the identity operator. Further, the problem (1.2) was introduced and studied by Agarwal and Verma [2] if $E = G \equiv I, g_i - p_i = 0$ for $i = 1, 2$, and operators $u(\lambda_1) = u$ and $x(\lambda_1) = x$ for all $\lambda_1 \in \Lambda_1$, $v(\lambda_2) = v$ and $y(\lambda_2) = y$ for all $\lambda_2 \in \Lambda_2$, and $N^i(u, v, \lambda_i) = N^i(u, v)$ for all $(u, v, \lambda_i) \in \mathbb{X}_1 \times \mathbb{X}_2 \times \Lambda_i$ and $i = 1, 2$ in (1.2).

Example 1.2. If $S : \mathbb{X}_1 \times \Lambda_1 \to \mathbb{X}_1$ and $T : \mathbb{X}_2 \times \Lambda_2 \to \mathbb{X}_2$ are single-valued operators, then for each fixed $\lambda_i \in \Lambda_i$ $(i = 1, 2)$, the problem (1.2) reduces to finding $(u(\lambda_1), v(\lambda_2)) \in \mathbb{X}_1 \times \mathbb{X}_2$ such that

$$\begin{cases} 0 \in N^1(u(\lambda_1), T_{\lambda_2}(v), \lambda_1) + M^1_{\lambda_1}((g_1 - p_1)_{\lambda_1}(u), E_{\lambda_1}(u)), \\ 0 \in N^2(S_{\lambda_1}(u), v(\lambda_2), \lambda_2) + M^2_{\lambda_2}((g_2 - p_2)_{\lambda_2}(v), G_{\lambda_2}(v)). \end{cases} \tag{1.3}$$

Example 1.3. If $\mathbb{X}_1 = \mathbb{X}_2 = \mathbb{X}, g_i - p_i = I$ $(i = 1, 2)$, $M^1_{\lambda_1}(u, v) = M^1_{\lambda_1}(u)$ for all $(u, v, \lambda_1) \in \mathbb{X} \times \mathbb{X} \times \Lambda_1$ and $M^2_{\lambda_2}(u, v) = M^2_{\lambda_2}(u)$ for all $(u, v, \lambda_2) \in \mathbb{X} \times \mathbb{X} \times \Lambda_2$, then the problem (1.3) is equivalent to the following system of generalized nonlinear parametric mixed quasi-variational inclusions: find $(u(\lambda_1), v(\lambda_2)) \in \mathbb{X} \times \mathbb{X}$ such that

$$\begin{cases} 0 \in N^1(u(\lambda_1), T_{\lambda_2}(v), \lambda_1) + M^1_{\lambda_1}(u), \\ 0 \in N^2(S_{\lambda_1}(u), v(\lambda_2), \lambda_2) + M^2_{\lambda_2}(v), \end{cases}$$

which was studied by Jeong and Kim [3] when $M^i_{\lambda_i}$ is H-accretive operators and N^i is a special case for $i = 1, 2$.

We remark that for appropriate and suitable choices of $N^i, M^i, S, T, E, G, g_i, p_i, A_i, \eta_i$ and $\mathbb{X}_i$ for $i = 1, 2$, it is easy to see that the problem (1.1) is a generalized version of some problems, which includes a number (systems) of (parametric) quasi-variational inclusions, (parametric) generalized quasi-variational inclusions, (parametric) quasi-variational inequalities, (parametric) implicit quasi-variational inequalities studied by many authors as special cases, see, for example, [4–23] and the references therein.

Further, the study of such types of problems is motivated by an increasing interest in the sensitivity analysis (or existence) of solutions for variational inclusion problems involving strongly monotone and relaxed cocoercive mappings under suitable second order and regularity assumptions have been carried out based on the general resolvent operator techniques by several researchers, see, for example, [1–6,8,11–23] and the references therein.

Moreover, it also generalizes the theory of multi-valued maximal monotone mappings and provides a general framework to examine convex programming and other variational inclusion problems. General resolvent operator techniques have been in use for a while and are being applied to a broad range of problems arising from model equilibria problems in economics, optimization and control theory, operations research, transportation network modelling, and mathematical programming.

Example 1.4 (*[24]*)**.** Let $\mathbb{X}$ be a real Hilbert space, and $M : dom(M) \subset \mathbb{X} \to \mathbb{X}$ be an operator on $\mathbb{X}$ such that M is monotone and $R(I + M) = \mathbb{X}$, Then based on the Yosida approximation $M_\rho = \frac{1}{\rho}(I - (I + \rho M)^{-1})$, for each given $u_0 \in dom(M)$, there exists exactly one continuous function $u : [0, 1) \to \mathbb{X}$ such that the following first-order evolution equation:

$$\begin{cases} u'(t) + Mu(t) = 0, \quad 0 < t < \infty, \\ u(0) = u_0, \end{cases} \tag{1.4}$$

where the derivative $u'(t)$ exists in the sense of weak convergence, that is,

$$\frac{u(t + h) - u(t)}{h} \rightharpoonup u'(t) \quad \text{as } h \to 0.$$

holds for all $t \in (0, \infty)$.

As Verma [20] pointed out "Among significant applications of the notion of A-maximal (m)-relaxed monotonicity, it seems that one can generalize the Yosida approximation and apply it to the solvability of evolution equations of the form (1.4)". It is well known that it is easier to solve the Yosida approximate evolution equation than Eq. (1.4). Further, regarding the results of Refs. [25–27], we know that the studied problem can be used to extend some duality results in nonsmooth optimization literature.

On the other hand, in [15], the author introduced a new concept of (A, η)-monotone operators, which generalizes the (H, η)-monotonicity and A-monotonicity in Hilbert spaces and other existing monotone operators as special cases, and some properties of (A, η)-monotone operators were studied and the resolvent operator associated with (A, η)-monotone operators was defined. At the same time, Verma [19] introduced the notion of (A, η)-maximal monotonicity (also referred