Optics and Lasers in Engineering 49 (2011) 210–221

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Sensitivity analysis applied to an improved Fourier-transform profilometry

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ARTICLE INFO

Article history:
Received 3 June 2010
Received in revised form
6 September 2010
Accepted 7 September 2010
Available online 16 October 2010

Keywords:
Sensitivity analysis
Uncertainty analysis
Fourier-transform profilometry
Calibration

ABSTRACT

Every estimation concerning model parameters has to deal with uncertainty, but its quantification is a complex task to solve, especially for the identification of the different uncertainty sources that affect the system. In this paper we provide an extensive uncertainty analysis on the calibration model of a Fourier-transform profilometry, studying the effect of the uncertainty parameters estimation on the final measuring result. The methods used for the classification are discrete derivatives and the global sensitivity analysis based on Monte Carlo simulations. The experimental results show that the uncertainty propagation of the system parameters to the output strongly depends on different system setups that may be chosen. This dependency is analysed and interpreted.

1. Introduction

Fringe projection techniques are very popular thanks to their possible application in a lot of fields, such as industrial inspection, manufacturing, computer and robot vision, reverse engineering and medical diagnostics. The main qualities of these profilometry methods are non-contact and full field measurement, low cost and speed in obtaining the 3D information [1–7]. One of the most used techniques is Fourier transform profilometry, which is based on the projection of a grid onto a surface and then viewed from another direction by a camera, which acquires the image [8]. The object topography deforms the fringe pattern: the corresponding image is acquired on the camera sensor plane and then processed to obtain depth information. The depth is extracted, through the Fourier-transform, from the phase difference between the grid projected on a reference plane and the same grid projected on the object surface. In comparison with other fringe techniques, which require more than one image for the 3D measurement, the advantages of the FTP are elaboration speed and need of only one deformed image [9–11]. On the other hand, it needs to resolve the projected grid lines individually, and consequently has a strong requirement on the pixel spatial resolution of the recording device. Moreover, FTP requires frequency domain filtering whose consequence is fine detail reduction and resolution loss [12,13].

Phase-to-depth conversion is possible by means of a suitable formula that depends on the geometric model of the acquisition system, i.e. its geometric parameters and the carrier frequency of the grid. Obtaining the correct depth distribution is possible only if the geometric parameters are known. Theoretically the task is easy to be achieved, but in practice the estimation of these parameters is quite complex: for example the relative position between the projector and the camera cannot be fixed without a certain degree of uncertainty, but also the evaluation of the carrier frequency of the grid and the reference plane position is affected by errors. A calibration procedure is consequently necessary to overcome these limitations. Calibration methods proposed in literature till now may be distinguished in three categories: model-based, polynomial and neural networks. The model-based methods try to define the correct phase-to-depth conversion formula by means of the indirect determination of the system parameters [14–18]. The polynomial methods otherwise use planes, placed at known positions, that are acquired and then processed to get phase distribution. The polynomial coefficients of the function that best fits the phase-depth data are estimated by a least squares algorithm to produce a calibration map [19–26]. Finally the last approaches apply several neural network methods to define the relationship between the inputs of the system (i.e. phase distribution) and the outputs of the system (i.e. depth distribution) with a black-box philosophy that is totally free from the geometric configuration [27].

In a previous paper we proposed a hybrid calibration method, between a model-based and a polynomial calibration process, based on an exhaustive geometric model, which describes the system with a generic relative position of the projector and the camera [28]. The parameters estimation is achieved by a minimization algorithm of the mean squared error between the nominal depth of some planes placed at well defined positions and the result of the conversion formula applied to the phase

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doi:10.1016/j.optlaseng.2010.09.016

obtained from the same planes. This calibration method was chosen because it has many advantages:

(a) The model is based on a well defined theory.
(b) It is based on a complete geometric model that has a physical correspondence with the real measurement setup; the main consequence is the direct comparison between the parameters estimation and the geometric quantities to avoid macroscopic errors.
(c) Both the geometric model and the pin-hole camera model include the main non-linearities, and then the errors due to the model simplifications are reduced.
(d) The calibration method is easy to apply.

A calibration method based on Mao's geometric model had already been presented in a previous work [29]. The aim of this paper is to improve the problem analysis, studying the effect of the uncertainty parameters estimation on the final result. The question is how the uncertainty concerning the geometric parameters of the system (input parameters) can be extended to depth estimation (measurement output). The answer may only come out from a sensitivity analysis of the calibration model. It must be noted that the results will be applicable for all the calibration methods based on Mao's geometric model (and its simplified version that corresponds to Takeda's model). The aim is the definition of a guide to uncertainty analysis about all the most common system setups to use in the design stage for the optimization of the measure uncertainty.

2. Measurement geometric model and calibration

Mao et al. [28] recently proposed a new phase mapping formula based on a complete geometric model of the projection profilometer, where the projector and the camera can be set freely as long as the full-field fringe pattern can be obtained. As it will be clarified later, the traditional FTP formula, which converts phase to depth, is just a special case of this general formula and is suitable only when the pupils of the camera and the projector are aligned [8]. The application of this geometric model is the main difference between our calibration method and most of the others proposed so far which are based on a simplified geometric model that do not consider geometric misalignment [29].

The widest formulation of the problem concerns a projector and a camera, which are not aligned, as shown in Fig. 1(top). The camera is translated along the three spatial directions with respect to the point G, which corresponds to the theoretical aligned position of the camera in the geometric setup proposed by Takeda and Mutoh [8]. The model is characterized by the carrier frequency of the projected grid and five geometric parameters: the distance $E_{20}O_1$ between the camera pupil and the reference plane R, the distance between the projector pupil E_1 and the point G, the vertical translation HE_{20} of the camera and finally the distance between points O and O' along both x and y directions. However, phase-to-depth conversion formula depends only on the first three parameters ($E_{20}O_1$, E_1G, HE_{20}) because, as declared by Mao himself, compared with the case in which the imaging axis crosses the projecting axis at the same point on the reference plane, moving the camera along the direction O_1O just causes the movement of the image location on the camera sensor array. This means that the translation of the camera along x and y directions is irrelevant to define the conversion formula between the extracted phase and the depth of the object, whereas the translation along z direction is essential. For this reason it is possible to face the problem with an equivalent two-dimensional geometric model, obtained by projecting the three-dimensional model along the O_1O direction on the plane E_1GO.

Fig. 1(bottom) shows the geometric model with the projector and the camera lying on the same plane, where the point E_2 is the correspondent to the point E_{20} along O_1O projection. Consequently the angle $H\hat{E}_1E_{20}$ and the distance E_1E_{20} between the pupils (Fig. 1(top)) respectively corresponds to the angle $G\hat{E}_1E_2$ and to the distance E_1E_2. The angle $G\hat{E}_1E_2$ and the distance E_1E_2 will respectively be named α and s. Thanks to these two geometric quantities, it is possible to quantify both the distance E_1G and the vertical translation GE_2. L is the distance between the entrance pupil of the imaging system and the reference plane R. In this geometric setup the optical axis of the projector lens still crosses the optical axis of the camera lens at point O on the reference plane. The angle between the optical axis of the projector and the camera is θ. D is an arbitrary point on the measured object and its coordinates are (x,y) on the reference plane. The points A and C represent the well-known effect of the fringe displacement due to the presence of the object instead of the reference plane.

When a sinusoidal fringe pattern is projected onto the reference plane, the captured fringe pattern on the reference plane may be expressed as [30]

$$I_1(x,y)=I_0\left\{1+\cos\left[2\pi fx\cos\theta\left(1-\frac{2x\sin^2\theta}{s\cos\alpha}\right)\right]\right\}, \tag{1}$$

where

$$\theta=\tan^{-1}\left(\frac{s\cos\alpha}{L+s\sin\alpha}\right),$$

and f is the spatial frequency of the grid projected on the plane normal to the projector optical axis and crossing the point O.

In the same way, the deformed fringe pattern can be expressed as

$$I_2(x,y)=I_0\left\{1+\cos\left[2\pi fx\cos\theta\left(1-\frac{2x\sin^2\theta}{s\cos\alpha}\right)-\psi(x,y)\right]\right\}, \tag{2}$$

where $\psi(x,y)$ is the phase distribution caused by the depth variation $h(x,y)$, which is [28] given as

$$\psi(x,y)=2\pi fCA\cos\theta,$$

and can be expressed through triangular similarity as [28]

$$\psi(x,y)=2\pi f\cos\theta\frac{BD}{L+s\sin\alpha-BD}\left(s\cos\alpha-\frac{xs\sin\alpha}{L-BD}\right). \tag{3}$$

The two signals $I_1(x,y)$ and $I_2(x,y)$ are processed with the well-known method proposed by Takeda to obtain the phase distribution $\psi(x,y)$ [8]. The method proposed by Takeda is one of the possible ways to remove the non-linear carrier frequency. Other methods are least-squares fitting and series expansion approach [31]. The new relationship between the phase distribution $\psi(x,y)$ and the depth distribution $h(x,y)$ is the core of this geometric model and its expression is [28]

$$BD=\frac{\phi_{DC}L(L+s\sin\alpha)}{2\pi f_0Ls\cos\alpha+L\phi_{DC}-\phi_Cs\sin\alpha}, \tag{4}$$

where L, s, α are geometric quantities and $f_0=f\cos\theta$ is the carrier frequency of the projected grid in the case of telecentric lens of the camera and the projector. From a theoretical point of view these parameters could be measured directly, but in practice, the accurate evaluation of these parameters with direct methods is extremely difficult. A calibration process is consequently required to overcome this problem. Moreover, ϕ_{DC} represents the phase difference between the object and the reference planes, and ϕ_C is the phase at point C on the reference plane. The phase difference ϕ_{DC} can be calculated in the same way of the traditional FTP