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Sensitivity analysis and optimization of vehicle–bridge systems based on combined PEM–PIM strategy

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ABSTRACT

This paper presents a new optimization method for coupled vehicle–bridge systems subjected to uneven road surface excitation. The vehicle system is simplified as a multiple rigid-body model and the single-span bridge is modeled as a simply supported Bernoulli–Euler beam. The pseudo-excitation method transforms the random surface roughness into the superposition of a series of deterministic pseudo-harmonic excitations, which enables convenient and accurate computation of first and second order sensitivity information. The precise integration method is used to compute the vertical random vibrations for both the vehicle and the bridge. The sensitivities are used to find the optimal solution, with vehicle ride comfort taken as the objective function. Optimization efficiency and computational accuracy are demonstrated numerically.

1. Introduction

Recently, dynamic analysis of coupled vehicle–bridge systems has received much attention because it relates to many engineering fields, e.g. interaction forces, dynamic effects and bridge design [1]. For such systems, it is well known that road irregularity is the most important factor causing random vibration. Analysis of such random vibration has long been regarded as very difficult. Optimum design is even more difficult but it is a matter of great concern and significance.

Fryba [2] pointed out that irregularities sometimes had a very appreciable random component for moving loads and for structures associated with transport, Silva [3] regarded the pavement roughness as a probabilistic model in order to investigate the dynamic behavior of a reinforced concrete highway bridge deck crossed by a heavy train and Xia [4] took the track irregularity as a major excitation when analysing the vibrations of train–bridge coupled systems. Vibration analysis of such coupled systems could be undertaken by conventional random vibration theory approaches [5], but unfortunately these are generally too complicated and inefficient. However very few research papers have been published which are relevant to efficient solution, which is the topic of this paper.

In general, difficulties mainly arise in the two areas identified in, respectively, the next two paragraphs.

Conventional methods for random vibration analysis take one or more sample curves as time-history input functions of road irregularity when estimating the random response of vehicle–bridge systems by means of time-history analysis and statistical processing [6–8]. This approach is obviously quite complicated and costly and so becomes particularly unacceptable for optimization problems because many re-analyses are required. Much work has been done to avoid costly computation, but unfortunately this results in limited accuracy and reliability because very few time-history curve samples are used in the analysis and optimization.

Any conventional numerical integration method, e.g. the Newmark method, requires the time-step size to be very small. This is because the vehicle is assumed to "jump" suddenly from one point to the next point at intervals, during which the magnitude of the contact force remains unchanged. However, in reality all wheels move continuously and every contact force may vary within each time step. Therefore significant errors will arise unless the time step is so small that it results in excessive computation costs.

To overcome the difficulties identified in the previous two paragraphs, the pseudo-excitation method (PEM) [9,10] has been applied to considerably simplify the solution of the dynamic equations by transforming the random surface roughness into the superposition of a series of deterministic pseudo-harmonic surface unevennesses. The precise integration method (PIM) [11,12] has also been extended to simulate the continuous variation within each time step of the magnitudes and positions of the contact

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forces. Hence the uniformly modulated, multi-point, different-phase, non-stationary random excitations of the road acting on the wheels are transformed into a deterministic pseudo-excitation vector by using extended PEM with the phase-lags between the wheels taken into account, so that the solution can be obtained efficiently by means of PIM.

In general, the dynamic optimization is efficient based on sensitivity information, but sensitivity analysis of random dynamic problems is more difficult. It is known that the derivatives of eigenvalues and eigenvectors have been studied by many scholars [13–15]. The MISA method [16] was proposed for optimizing structures subjected to dynamic stress and displacement constraints. The least-squares iteration method has been used to compute the first-order eigenvalue sensitivity and sometimes the second-order sensitivity [17]. Sensitivity analysis of structures with transient dynamic load under stress constraints was investigated by Durbin [18]. The optimization design of a wing structure excited by random gust loads was investigated by Rao [19] with the sensitivity formulas of mean-square responses being derived. A matrix perturbation method [20] for sensitivity analysis of structural dynamic responses was developed. Some methods for dynamic optimization designs of structures have been discussed and compared [21]. Unfortunately, relatively few papers relevant to the algorithm and application of second order flexibility, particularly with respect to random vibration, can be found in the literature. In fact, the first-order sensitivity analysis only indicates the local optimal direction, whereas the second-order sensitivity analysis is sometimes quite useful for obtaining the global optimal solution. Although the Hessian matrix is a well-known means for calculating general second-order flexibilities, the enormous effort required by it seriously restricts its practical applications. These sensitivity analysis method are both based on conventional inefficient random vibration methods, and are naturally ineffective as well.

Our paper presents a new method for first and second order sensitivity analyses of structural random responses, which is based on PEM–PIM and on previous optimality research work [22]. The method replaces the right-hand side random excitation of the random equations of motion by pseudo-excitations, in order to enable the convenient and accurate derivation of various first and second order sensitivity formulae. Numerical examples demonstrate the correctness and high efficiency of this new method, which is then used in the optimum design of a bus, with ride comfort as the objective function.

2. Vehicle–bridge equations of motion based on coupled vibrator-Euler beam model

In practice, the optimal control of vehicle vibration becomes the optimization of acceleration, displacement and ride comfort of the suspension system for coupled vehicle–bridge systems. Therefore the linear model of a coupled vehicle–bridge system shown in Fig. 1 is used, which accords with the usual practice of facilitating the investigation of complex vehicle–bridge random vibration problems by simplifying the vehicle to become a moving multiple rigid-body oscillator model and by representing the single-span bridge as a simply supported elastic Bernoulli–Euler beam of length L. The vehicle model has many DOF, so for convenience, a 2-D vehicle model with two axles and four DOF is used, with dynamic behavior described by vertical and pitching motions. The vehicle is assumed to travel along the x axis at a constant velocity v and yaw motion is ignored because it affects vehicle ride comfort negligibly [23].

D'Alembert's principle gives the equations of motion of the vehicle model of Fig. 1 as Eq. (1), in which the system parameters are: m_c is the mass of the vehicle body; J is the rotational inertia of the vehicle about its y axis; m_f and m_r are the masses of the wheel axles

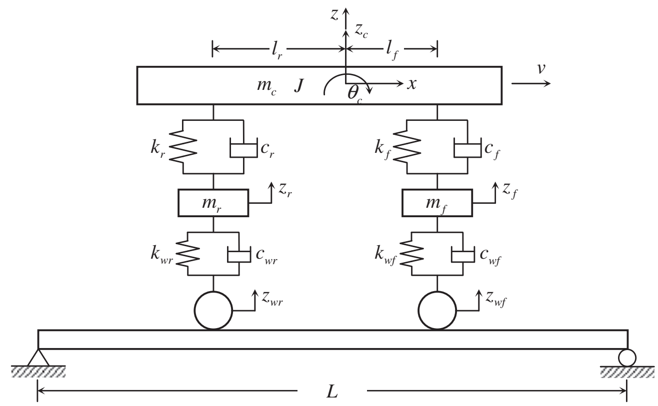

Fig. 1. Coupled vehicle–bridge model.

(with f representing front and r representing rear); k_f and k_r are the corresponding stiffness coefficients; c_f and c_r are the corresponding damping coefficients; k_{wf} and k_{wr} are the tire stiffness coefficients; c_{wf} and c_{wr} are the tire damping coefficients and; l_f and l_r denote the distances between the body-center and the front or rear axle.

$$\mathbf{M}_c\ddot{\mathbf{z}}_c + \mathbf{C}_c\dot{\mathbf{z}}_c + \mathbf{K}_c\mathbf{z}_c = \mathbf{F}_c(t) \quad (1)$$

Because the wheels are assumed to remain in perfect contact with the track at all times, the constraint conditions between the wheel set and bridge can be described as

$$z_{wi} = z_{bi} + R(x_i)(i = f, r) \quad (2)$$

in which z_{bi} is the vertical displacement of the bridge and $R(x_i)$ represents the road displacement roughness at this set.

The bridge is discretized using Bernoulli–Euler beam elements [24], and the resulting dynamic equation has the form

$$\mathbf{M}_b\ddot{\mathbf{z}}_b + \mathbf{C}_b\dot{\mathbf{z}}_b + \mathbf{K}_b\mathbf{z}_b = \mathbf{F}_b(t) \quad (3)$$

where $\mathbf{M}_b$ is the consistent mass matrix, and $\mathbf{C}_b$ is the Rayleigh damping matrix. The excitation vector $\mathbf{F}_b(t)$ is usually complicated, and can be decomposed into two terms, namely the deterministic loads $\mathbf{F}_{bp}(t)$ due to vehicle gravity and the time-variable loads $\mathbf{F}_{bR}(t)$ due to road irregularity, where

$$\mathbf{F}_{bp}(t) = \left(\frac{l_r m_c}{l_f + l_r} + m_f\right)\mathbf{N}_{tf} + \left(\frac{l_f m_c}{l_f + l_r} + m_r\right)\mathbf{N}_{tr} \quad (4)$$

$$\begin{aligned}\mathbf{F}_{bR}(t) = {} & k_{wf}(z_f - z_{wf})\mathbf{N}_{tf} + c_{wf}(\dot{z}_f - \dot{z}_{wf})\mathbf{N}_{tf} \\ & + k_{wr}(z_r - z_{wr})\mathbf{N}_{tr} + c_{wr}(\dot{z}_r - \dot{z}_{wr})\mathbf{N}_{tr}\end{aligned} \quad (5)$$

Here $\mathbf{N}_{tf}$ and $\mathbf{N}_{tr}$ are the shape function vectors of conventional FEM, which distribute the vehicle loads between the nodes of the beam element.

Substituting Eq. (2) into Eqs. (1) and (3) gives the equation of motion for the coupled vehicle–bridge system as

$$\mathbf{M}\ddot{\mathbf{z}} + \mathbf{C}\dot{\mathbf{z}} + \mathbf{K}\mathbf{z} = \mathbf{F}(t) \quad (6)$$

where: the vector $\mathbf{z}$ is the coupled displacement vector $\mathbf{z} = \{z_c \quad \theta \quad z_f z_r z_{b1} \cdots z_{bn}\}$ and; $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ represent respectively the coupled mass, stiffness and damping matrices.

The coupled excitation vector $\mathbf{F}(t)$ is composed from the deterministic loads $\mathbf{F}_p(t)$ and the time-variable loads $\mathbf{F}_R(t)$, given by

$$\mathbf{F}_p(t) = \left(\frac{l_r m_c}{l_f + l_r} + m_f\right)\begin{Bmatrix}0\\0\\0\\0\\\mathbf{N}_{tf}\end{Bmatrix} + \left(\frac{l_f m_c}{l_f + l_r} + m_r\right)\begin{Bmatrix}0\\0\\0\\0\\\mathbf{N}_{tr}\end{Bmatrix} \quad (7)$$