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The sensitivity analysis of propagator for path independent quantum finance model

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ABSTRACT

Quantum finance successfully implements the imperfectly correlated fluctuation of forward interest rates at different maturities, by replacing the Wiener process with a two-dimensional quantum field. Interest rate derivatives can be priced at a more realistic value under this new framework. The quantum finance model requires three main ingredients for pricing: the initial forward interest rates, the volatility of forward interest rates, and the correlation of forward interest rates at different maturities. However, the hedging strategy only focused on fluctuation of forward interest rates. This hedging method is based on the assumption that the propagator, the covariance of forward interest rates, has an ergodic property. Since inserting the propagator is the main characteristic that distinguishes quantum finance from the Libor market model (LMM) and the Heath, Jarrow and Morton (HJM) model, understanding the impact of propagator dynamics on the price of interest rate derivatives is crucial. This research is the first step in developing a hedge strategy with respect to the evolution of the propagator. We analyze the dynamics of the propagator from Libor futures data and the integrated propagator from zero-coupon bond rate data. Then we study the sensitivity of the implied volatility of caplets and swaptions according to the three dominant dynamics of the propagator, and the change of the zero-coupon bond option price according to the two dominant dynamics of the integrated propagator.

1. Introduction

The option for interest rates is a more complex instrument than the equity option because it needs to handle a multitude of random variables, the future interest rates observed in the market, at every instance. Future interest rates evolve randomly, and they are highly correlated with each others. However, the market standard interest rate models, HJM model [1] and LMM [2,3], are built on the foundation that the random variable is perfectly correlated. Thus, the option price for interest rates from these models is overvalued to the real market price [4]. Furthermore, these models allow the possibility to perfectly hedge the short term interest rate instruments with long term ones. Since this is unacceptable in the real world, the complicated multi-factor model, which shows that the interest rates at any instance are driven by more than one random variable, is frequently used [5]. However, as the number of factors grows, the solution for the option price becomes severely complicated [5].

One celebrated idea to handle this problem is the stochastic string model which is suggested by Sornette and others [6,7]. This model assumes the interest rates as a string and adopts the concept of Brownian sheet. Then, it can successfully reduce the number of factors, while it can generate the realistic smooth correlation structure between different time to maturity interest rates [8]. It also provides the more realistic hedging strategy which suggests that the similar maturity interest

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rates must be selected as hedging product to minimize the given interest rate risk. It also solves the possible kink problem in interest rate term structure involved in multi-factor HJM model and LMM. Through the empirical studies using well calculated analytic formula for price of caps and swaptions, the usefulness and sensitivity of this model are successfully analyzed [9,10]. String model is still referred to recent papers to compare their results, and it keeps advanced in many finance papers [11–13].

Quantum finance introduces a new framework to calculate and hedge the interest rate derivatives. Baaquie created Lagrangian with a two-dimensional quantum field and successfully calibrated the correlation structure of the interest rates at different future times [14,15]. The string model also did the correlation structure. With this framework, he also developed new formulas for path independent options and a path-dependent option such as the barrier option for interest rates [16–18]. He also suggested a new numerical algorithm to price the American option for interest rates in a quantum finance framework [19]. His empirical study showed that the expected price of swaptions from quantum finance formalism is better than the standard market model [4]. In addition, he showed that a caplet cannot be perfectly hedged with Libor futures unless their maturity is exactly the same [20,21]. Furthermore, the empirical study showed that the dynamics of the simple interest rate market (Libor market) and the bond market, which is represented by a zero-coupon yield curve (ZCYC), do not share the same dynamics [22]. Even though some of the results from quantum finance had been already known from the papers based on string model, quantum finance has its own value because it draws the results from its own framework, and it is a promising framework to calculate more exotic options [18].

The strength of quantum finance can be summarized as the usage of a suitable propagator that connotes the covariance between interest rates at different futures. This propagator is assumed to be an ergodic process in quantum finance. However, for practical purposes, the evolution of the covariance structure must be considered to price or hedge the interest rate derivatives [23]. Therefore, a sensitivity analysis of the propagator for numerous option prices for interest rates can contribute to the progress of quantum finance, as a support in developing the new hedge technique to prepare a deviation of the propagator from the initial observation. With this stimulus, we study the impact of propagator dynamics on the implied volatility of caplets, swaptions, and the impact of integrated propagator dynamics on the price of a coupon bond option with a ceteris paribus assumption.

This paper is organized as follows. In Section 2, we review the quantum finance model and how to price interest rate derivatives using volatility expansion. Caplets and swaption are priced using extended Libor market model with second order volatility expansion, and a coupon bond option is priced using quantum bond market model with fourth order volatility expansion. Section 3 shows how we apply the principal component analysis (PCA) to study the dynamics of the propagator from the simple interest rate market and the integrated propagator from the bond market. We also inform how to gather the data set using the information on the composition of the tickers for Libor futures in Bloomberg and zero-coupon yield curve data in the Reuters Datastream terminal. In Section 4, how the propagator dynamics affects the implied volatility of caplets and swaptions under extended LMM, and how the integrated propagator dynamics affects the price of coupon bond option under extended HJM model are discussed. Finally, we briefly discuss the time series of innovation. In Section 5, our conclusion is drawn with a brief summary.

2. Quantum field theory model

We will review two famous models for interest rate derivatives, the HJM model and the LMM. The HJM model uses the forward interest rates $f(t,x)$ and the volatility of $f(t,x)$ as the main ingredient. $f(t,x)$ is the interest rate for an instantaneous loan at a future time x at a fixed time t. Here, $x > t$. Therefore, the value of a risk free zero-coupon bond $B(t,T)$, which pays a unit dollar at future time $T > t$, can be calculated from $f(t,x)$ as

$$B(t,T) = \exp\left(-\int_t^T \mathrm{d}x f(t,x)\right). \tag{1}$$

The zero-coupon bond is also represented by the market observable zero-coupon yield curve (ZCYC), $Y(t,T)$,

$$B(t,T) = \frac{1}{\left(1+\frac{1}{k}Y(t,T)\right)^{k[T-t]}}. \tag{2}$$

Here, k is the number of interest rates paid out in a year. For the semi-annual payment, $k=2$, in a treasury bond market. In this paper, the dimension of time is fixed as a year unless noted otherwise. The value of the forward bond, which is issued at some future time t_* and matures at time T, is

$$F(t_0,t_*,T) = B(t_*,T) = \exp\left(-\int_{t_*}^T \mathrm{d}x f(t,x)\right). \tag{3}$$

Here, t_0 is the present time. In the HJM model, the dynamics of $f(t,x)$ is given as a function of white noise

$$\frac{\partial f(t,x)}{\partial t} = \alpha(t,x) + \sigma(t,x)R(t). \tag{4}$$