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Sensitivity analysis of inefficient units in data envelopment analysis

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ABSTRACT

One important issue in DEA which has been studied by many DEA researchers is the sensitivity of the results of an analysis to perturbations in the data.

This paper develops a procedure for performing a sensitivity analysis of the inefficient decision making units (DMUs). The procedure yields an exact "*Necessary Change Region*" in which the efficiency score of a specific inefficient DMU changes to a defined efficiency score.

In what follows, we identify a new frontier, and prove the efficiency score of each arbitrary unit on it which is defined as the efficiency score.

1. Introduction

Data envelopment analysis (DEA) introduced by Charnes et al. [1] (CCR) and extended by Banker et al. [2] (BCC), is a useful method to evaluate the relative efficiency of multiple-input and multiple-output units based on the data observed. The sensitivity analysis has received great attention from researchers in recent years and so much research has been carried out in this regard. Sensitivity analysis in DEA has been deliberated from various points of view.

One important issue in DEA which has been studied by many DEA researchers, is the sensitivity analysis of a specific decision making unit (DMU) which is under evaluation [3–7]. Another type of DEA sensitivity analysis is based on the super-efficiency DEA approach in which the DMU under evaluation is not included in the reference set [8–12]. Charnes et al. developed a super-efficiency DEA sensitivity analysis technique for the situation where simultaneous proportional change is assumed in all inputs and outputs for a specific DMU under consideration [13,14].

DEA sensitivity analysis methods that we have just reviewed are all developed for the situation where data variations are only applied to the efficient DMU under evaluation and the data for the remaining DMUs are assumed fixed.

While the sensitivity analysis of an efficient unit's classification has been extensively studied, the issue of an inefficient unit's estimation and classification seems to be ignored.

This paper focusses on inefficient sensitivity analysis DMUs. So, the aim is to research ways to improve the inefficient units using another strategy, in addition to the evaluation of DMUs and classifying them into efficient and inefficient.

The improvement is usually possible, but sometimes, reaching to the efficiency frontier and achieving the score 1 in efficiency by inefficient units are really impossible. Our objective is to reach to the efficiency score of those inefficient units whose efficiency score is less than a fixed constant α to α. (This constant is usually close to 1 and is defined by the manager). It means that if we suppose the efficiency score of inefficient unit to be θ_o^* and $\theta_o^* < \alpha < 1$, then after these variations, it will meet the efficiency score of α, and an improvement of $\alpha - \theta_o^*$ in efficiency is obtained.

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The variations region of every inefficient unit is called "*Necessary Change Region*". In what follows, some new frontiers are defined and with the help of some theorems, we will prove that the efficiency score of each unit of the new frontier is α. In fact, as the efficiency score of all points on the main frontier is supposed to be 1, the efficiency score on the new frontiers is α.

This paper proceeds as follows. Section 2 discusses the basic DEA models. Section 3 develops a proposed method for finding the "*Necessary Change Region*". Section 4 provides a numerical example and finally, conclusions are given in Section 5.

2. Background

Data Envelopment Analysis (DEA) is a technique that has been used widely in the literature of the supply chain management. This non-parametric, multi-factor approach enhances our ability to capture the multi-dimensionality of performance discussed earlier. More formally, DEA is a mathematical programming technique for measuring the relative efficiency of decision making units (DMUs) where each DMU has a set of inputs used to produce a set of outputs.

Consider DMU_j, $(j = 1, \ldots, n)$, where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $X_j = (x_{1j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, \ldots, y_{sj})$ respectively, and let $X_j \geq 0$ and $X_j \neq 0$ and $Y_j \geq 0$ and $Y_j \neq 0$.

The production possibility set T_c is defined by:

$$T_c = \left\{ (X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j,\ Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0,\ j = 1, \ldots, n \right\}.$$

The above definition implies that the CCR model is as follows:

$$\begin{aligned} \min\quad & \theta \\ \text{s.t.}\quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s \\ & \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{aligned} \tag{1}$$

Moreover, the production possibility set T_v is defined by:

$$T_v = \left\{ (X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j,\ Y \leq \sum_{j=1}^{n} \lambda_j Y_j,\ \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0,\ j = 1, \ldots, n \right\}.$$

The above definition implies that the BCC model is as follows:

$$\begin{aligned} \min\quad & \theta \\ \text{s.t.}\quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{aligned} \tag{2}$$

In addition, the multiplier forms of the CCR, BCC models are:

CCR model

$$\begin{aligned} \max\quad & \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t.}\quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & v_i \geq 0, \quad i = 1, \ldots, m \\ & u_r \geq 0, \quad r = 1, \ldots, s. \end{aligned} \tag{3}$$