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Belief function combination and conflict management

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Abstract

Within the framework of evidence theory, data fusion consists in obtaining a single belief function by the combination of several belief functions resulting from distinct information sources. The most popular rule of combination, called Dempster's rule of combination (or the orthogonal sum), has several interesting mathematical properties such as commutativity or associativity. However, combining belief functions with this operator implies normalizing the results by scaling them proportionally to the conflicting mass in order to keep some basic properties. Although this normalization seems logical, several authors have criticized it and some have proposed other solutions. In particular, Dempster's combination operator is a poor solution for the management of the conflict between the various information sources at the normalization step. Conflict management is a major problem especially during the fusion of many information sources. Indeed, the conflict increases with the number of information sources. That is why a strategy for re-assigning the conflicting mass is essential. In this paper, we define a formalism to describe a family of combination operators. So, we propose to develop a generic framework in order to unify several classical rules of combination. We also propose other combination rules allowing an arbitrary or adapted assignment of the conflicting mass to subsets.

Keywords: Data fusion; Evidence theory; Rules of combination; Conflict

1. Introduction

Information fusion has been the object of much research over the last few years [1–11]. Generally, it is based on the confidence measure theory (possibility theory, evidence theory, probability theory and fuzzy set theory) and has the advantage of:

- using redundant information,
- using the complementarity of the available information,
- achieving more reliable information,
- improving the decision making.

Data fusion is used in many application fields, such as multi sensor fusion [12,13], image processing and analysis [4–7,11,14,15], classification [16–18] or target tracking [19]. It takes into account heterogeneous information (numerical or symbolic) which is often imperfect (imprecise, uncertain and incomplete) and modeled by means of sources which have to be combined or aggregated. In the framework of evidence theory, information fusion relies on the use of a combination rule allowing the belief functions for the different propositions to be combined. The basic rule of combination is Dempster's rule of combination (orthogonal sum). It needs a normalization step in order to preserve the basic properties of the belief functions. In [20], Zadeh has underlined that this normalization involves counter-intuitive behaviours. In order to solve the problem of conflict management, Yager [21], Dubois [22] and Smets [23] and more recently Murphy [24] have proposed other combination rules. However, these rules have more or less satisfactory behaviours. In particular, Dubois' rule or Yager's rule of combination hold that the conflicting mass must be distributed over all subsets. Smets proposes that the conflicting mass results from the non-exhaustivity of the frame of discernment. We propose another approach, in which we define a

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generalized framework for the fusion of information sources by means of a generic axiomatic. This framework enables a large family of combination rules to be obtained.

This paper is organized as follows. The basic concepts of evidence theory are first briefly introduced (Section 2) including the problem of conflict in Dempster's rule of combination. In Section 3, we define the generic framework allowing classical combination operators to be unified and we propose a family of new combination rules. Finally, some methods to determine weighting factors for the conflicting mass distribution process for each proposition implied in the conflict are proposed (Section 3.3.2). Tests are given in Section 4.

2. Background

Evidence theory is initially based on Dempster's work [25] concerning lower and upper probability distribution families. From these mathematical foundations, Shafer [26] has shown the ability of the belief functions to modelize uncertain knowledge. The usefulness of belief functions, as an alternative to subjective probabilities, was later demonstrated axiomatically by Smets [27] and Smets and Kennes [28] with the *transferable belief model* (TBM) giving a clear and coherent interpretation of the underlying concept of the theory.

2.1. Knowledge model

Evidence theory first supposes the definition of a set of hypotheses Θ called the frame of discernment, defined as follows:

$$\Theta = \{H_1, \dots, H_n, \dots, H_N\}. \tag{1}$$

It is composed of N exhaustive and exclusive hypotheses. From the frame of discernment Θ, let us denote 2^Θ, the power set composed with the 2^N propositions A of Θ:

$$2^\Theta = \{\emptyset, \{H_1\}, \{H_2\}, \dots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \dots, \Theta\}. \tag{2}$$

A key point of evidence theory is the *basic belief assignment* (bba). The mass of belief in an element of Θ is quite similar to a probability distribution, but differs by the fact that the unit mass is distributed among the elements of 2^Θ, that is to say not only on the singletons H_n in Θ but on composite hypotheses too. The belief m_j assigned to an information source S_j is thus defined by

$$m_j : 2^\Theta \to [0, 1]. \tag{3}$$

This function verifies the following properties:

$$m_j(\emptyset) = 0, \tag{4}$$

$$\sum_{A \subseteq \Theta} m_j(A) = 1. \tag{5}$$

The mass $m_j(A)$ represents how strongly the evidence supports A which, in the case of a disjunction of hypotheses, has not been assigned to a subset of A because of insufficient information. This mass can be re-assigned more precisely to the subsets of A if additional information is available. Each subset $A \subseteq \Theta$ such as $m_j(A) > 0$ is called a focal element of m. Let us denote $\mathscr{F}_j$ the set of the focal elements associated to a belief function m_j. From this bba, a belief function Bel_j and a plausibility function Pl_j are defined, respectively, as

$$Bel_j(A) = \sum_{B \subseteq A} m_j(B) \tag{6}$$

and

$$Pl_j(A) = \sum_{A \cap B \neq \emptyset} m_j(B). \tag{7}$$

The quantity $Bel_j(A)$ can be interpreted as a measure of one's belief that hypothesis A is true. The plausibility $Pl_j(A)$ can be viewed as the total amount of belief that could be potentially placed in A. Note that functions m_j, Bel_j and Pl_j are in one-to-one correspondence [26], and can be seen as three facets of the same piece of information.

In evidence theory, one of the main difficulties lies in modelling the knowledge of the problem by initializing the belief functions m_j as well as possible. Generally, the model depends on the application. In [29], Appriou proposes two models in order to manage the uncertain learning in the framework of evidence theory. These models are consistent with the Bayesian approach when the belief mass is only allocated to singletons. Other models, also based on likelihood functions, have been proposed [30–32]. Another method based on the use of a neighbourhood information was introduced by Denoeux [17,18,33,34].

2.2. Dempster's rule of combination

In the case of imperfect data (uncertain, imprecise and incomplete), fusion is an interesting solution to obtain more relevant information. Evidence theory offers appropriate aggregation tools. From the basic belief assignment denoted m_j obtained for each information source S_j, it is possible to use a combination rule in order to provide combined masses synthesizing the knowledge of the different sources. These belief masses can then be used by a decision process with the benefit of the whole knowledge contained in the belief functions given by each source.

Dempster's rule of combination [26] is the first one defined within the framework of evidence theory. Using the rule implies that the independence condition for the