



A sensitivity analysis for the determination of unknown thermal coefficients through a phase-change process with temperature-dependent thermal conductivity[☆]

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ABSTRACT

In Tarzia, Int. Comm. in Heat and Mass Transfer, 25 (1998), 139–147, explicit formulas for the simultaneous determination of unknown thermal coefficients of a semi-infinite material through a phase-change process with temperature-dependent thermal conductivity were obtained. Moreover, ten different cases were studied: four cases of free boundary problems (i.e. Stefan-like problems) and six cases of moving boundary problems (i.e. inverse Stefan-like problems).

The goal of this paper is to obtain a numerical sensitivity analysis of the mentioned ten cases for the simultaneous determination of unknown thermal coefficients and to determine the coefficients which are more sensitive with respect to the given parameters. We show numerical result for the aluminum.

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1. Introduction

Heat transfer problems with a phase-change such as melting and freezing have been studied in the last century due to their wide scientific and technological applications. A review of a long bibliography on moving and free boundary problems for phase-change materials (PCM) for the heat equation is shown in [16].

We consider the following solidification problem for a semi-infinite material with an over specified condition on the fixed face $x=0$ [1,3,4,7]:

$$\begin{cases} \text{i) } \rho c T_t(x, t) = (k(T) T_x(x, t))_x, & 0 < x < s(t), \quad t > 0 \\ \text{ii) } T(0, t) = T_o < T_f, & t > 0 \\ \text{iii) } k(T_o) T_x(0, t) = \frac{q_o}{\sqrt{t}}, & t > 0, \quad q_o > 0 \\ \text{iv) } T(s(t), t) = T_f, & t > 0 \\ \text{v) } k(T_f) T_x(s(t), t) = \rho h \dot{s}(t), & t > 0 \end{cases} \quad (1)$$

where $T(x, t)$ is the temperature of the solid phase, $\rho > 0$ is the density of mass, $h > 0$ is the latent heat of fusion by unity of mass, $c > 0$ is the specific heat, $x = s(t)$ is the phase-change interface, T_f is the phase-change temperature, T_o is the temperature at the fixed face $x=0$ and

q_o is the coefficient that characterizes the heat flux at $x=0$ given by Eq. (1iii), which must be obtained experimentally through a phase-change process [2]. We suppose that the thermal conductivity has the following expression [5]:

$$k = k(T) = k_o \left[1 + \beta(T - T_o) / (T_f - T_o) \right], \quad \beta \in \mathbb{R}. \quad (2)$$

Let $\alpha_o = k_o / \rho c$ be the coefficient of the diffusivity at the temperature T_o . We observe that if $\beta=0$, the problem (1) becomes the classical one-phase Lamé-Clapeyron-Stefan problem with an overspecified condition at the fixed face $x=0$, and for this problem the corresponding simultaneous determination of thermal coefficients was studied in [13,14]. The phase-change process with temperature-dependent thermal coefficient of the type (2) was firstly studied in [5]. Other papers related to determination of thermal coefficients are [8,10,11,17–20].

The solution to problem (1) is given by [5,15]:

$$\begin{cases} \text{i) } T(x, t) = T_o + \frac{(T_f - T_o)}{\Phi(\lambda)} \Phi(\eta), & \eta = \frac{x}{2\sqrt{\alpha_o t}}, \quad 0 < \eta < \lambda \\ \text{ii) } s(t) = 2\lambda\sqrt{\alpha_o t} \end{cases} \quad (3)$$

where $\Phi = \Phi(x) = \Phi_o(x)$ is the modified error function, for a given $\delta > -1$, the unique solution to the following boundary value problem in variable x , i.e:

$$\begin{cases} \text{i) } [(1 + \delta \Phi'(x)) \Phi'(x)]' + 2x\Phi'(x) = 0, & x > 0, \\ \text{ii) } \Phi(0^+) = 0, & \Phi(+\infty) = 1 \end{cases} \quad (4)$$

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Nomenclature

<i>c</i>	Specific heat, J/ (kg°C)
<i>h</i>	Latent heat of fusion by unit of mass, J/kg
<i>k</i>	Thermal conductivity, W/(m°C)
<i>q₀</i>	Coefficient that characterizes the heat flux at <i>x</i> = 0, kg/s ^{5/2}
<i>s</i>	Position of the free or moving front, m
Ste	The Stefan number defined by Eq. (9), dimensionless
<i>t</i>	Time, s
<i>T</i>	Temperature, °C
<i>x</i>	Spatial coordinate, m

Greek symbols

α	Diffusivity coefficient, m ² /s
β	Coefficient that characterizes the thermal conductivity in Eq. (2), dimensionless
δ	Coefficient that characterizes the differential Eq. (4i), dimensionless
η	Similarity variable defined by Eq. (3), dimensionless
λ	Coefficient that characterizes the free boundary in Eq. (3ii), dimensionless
ρ	Density, kg/m ³
σ	Coefficient that characterizes the moving boundary in Eq. (3iibis), m/s ^{1/2}

Subscripts

<i>f</i>	Fusion
<i>o</i>	Initial in time or in space

and the unknown thermal coefficients must satisfy the following system of equations [15]:

$$\beta - \delta \Phi(\lambda) = 0 \tag{5}$$

$$[1 + \delta \Phi(\lambda)] \frac{\Phi'(\lambda)}{\lambda \Phi(\lambda)} - \frac{2h}{c(T_f - T_o)} = 0 \tag{6}$$

$$\frac{\Phi'(0)}{\Phi(\lambda)} - \frac{2q_o}{(T_f - T_o)\sqrt{k_o \rho c}} = 0. \tag{7}$$

For the particular case $\delta = 0$ we have that $\Phi(x) = \text{erf}(x)$ is the error function, which is defined by:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du. \tag{8}$$

We remark that if problem (1) is a free boundary problem (this case can be considered as a Stefan problem) with an overspecified condition on the fixed face $x=0$, then the coefficient $\lambda > 0$ is an unknown coefficient. On the other hand, if problem (1) is a moving boundary problem (this case can be considered as an inverse Stefan problem) with an overspecified condition on the fixed face $x=0$, then the phase-change interface will be given by

$$s(t) = 2\sigma\sqrt{t} \tag{3iibis}$$

where σ must be obtained experimentally ($\sigma = \lambda\sqrt{\alpha_o}$) through a phase-change process [2,14].

When the coefficient $\delta = 0$, the corresponding determination of formulas for one or two unknown thermal coefficients were obtained in [13,14] and the numerical-experimental determination was given in [2]. When the coefficient $\delta \neq 0$ is given, the corresponding problem was analyzed in [15]; in this case, the necessary and sufficient conditions on the data were obtained in order to ensure the existence of the solution.

The goal of the present paper is to make the sensitivity analysis of the free and moving boundary problems, analyzed in [15]. For a one-phase Stefan problem, the temperature $T(x,t)$, the free boundary interface $s(t)$ (i.e. the coefficient λ , defined in Eq. (3ii), is also an unknown coefficient) and the following parameters in four different cases:

FB:	i) λ, β, k_o	ii) λ, β, ρ	iii) λ, β, h	iv) λ, β, c
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were determined in [15]. For a one-phase inverse Stefan problem (i.e. the interface $s(t)$ is given by Eq. (3iibis) for a given $\sigma > 0$), the temperature $T(x,t)$ and the following parameters in six cases:

MB:	i) β, k_o, ρ	ii) β, k_o, c	iii) β, k_o, h	iv) β, ρ, c	v) β, ρ, h	vi) β, c, h
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were also determined in [15].

The explicit formulas corresponding to the ten cases for the unknown thermal coefficients were summarized in [15] (see Table 1). For cases FB (iii and iv) and MB (ii, iv, v and vi) the data must satisfy certain restrictions in order to obtain the solution of the corresponding thermal problem. These restrictions, called R_1, R_2, R_3 and R_4 in [15], are the following:

$$\frac{(T_f - T_o)\Phi'(0)}{2q_o} \sqrt{k_o \rho c} < 1 \tag{R1}$$

$$\frac{(T_f - T_o)k_o \rho h}{2q_o^2} < 1 \tag{R2}$$

$$\frac{\rho \sigma h}{q_o} < 1 \tag{R3}$$

Table 1
Left and right normalized sensitivities in the four cases of free boundary problems.

Case number	Unknown Coefficients	δ	k_o	ρ	c	h
1	λ	0.008	0.008	-	0	0.48
	β	1.01	1.01	-	0	0.46
	k_o	-0.015	-0.015	-	-1.01	-0.99
2	λ	0.008	0.008	0	-	0.48
	β	1.01	1.01	0	-	0.46
	ρ	-0.015	-0.015	-1.01	-0.99	-0.082
3	λ	0.016	0.016	0.52	0.52	-
	β	1.02	1.02	0.5	0.49	-
	h	-0.016	-0.016	-1.1	-1.07	-0.081
4	λ	-0.084	-0.084	-5.85	-6.08	-5.85
	β	0.92	0.92	-5.52	-5.78	-5.52
	c	-0.19	-0.19	-12.4	-12.1	-11.3

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