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Global sensitivity analysis of unreinforced masonry structure using high dimensional model representation

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ABSTRACT

Sensitivity Analysis (SA) has very important implications in terms of model assessment. It is an important part of reliability studies as well. This paper presents global SA using High Dimensional Model Representation (HDMR) on a mesoscale model of unreinforced masonry shear wall. The mesoscale model contains both geometric as well as material nonlinearity. Prior to performing global SA: (a) mesh sensitivity study in order to determine the optimum mesh size; and (b) experimental validation of the finite element simulation using the data available in the literature, are conducted. The ability of two major variations of HDMR, namely RS (Random Sampling)-HDMR and Cut-HDMR, for conducting global SA is explored, first by solving analytical problem and later by analyzing the mesoscale model of unreinforced shear wall. From the study, recommendations are made to obtain the sensitive parameters of an unreinforced masonry structure with minimal computational effort.

1. Introduction

Very often we encounter physical or mathematical models which encompass a large number of input variables. Henceforth, an essential part of model evaluation should include some performance assessment of the model in prediction of response, in case insufficient data regarding input parameters is available. This is usually done through sensitivity analysis (SA) and uncertainty analysis. The main purpose of SA is to estimate effects of each model input, independently or through cooperative efforts, on the model response and to identify the primary contributors to the output uncertainty [1,2]. Whereas, uncertainty analysis looks into the uncertainty in the model response given uncertainty in input space [1]. SA can be broadly categorized into two groups depending on the type of analysis conducted. When SA is performed by varying each input parameter keeping other input parameters constant, it is called local SA [3]. Whereas, global SA is performed over whole input domain and the interaction between input parameters is taken into account. If the model is non-linear in nature results obtained by local SA are not representative [4].

Recently application of global SA to structural systems has caught the attention of the research community [5–7]. Very few works on local SA of masonry have been reported so far [8–12]. Lourenco [8] conducted local SA of an unreinforced shear wall using finite element analysis both on mesoscale and macro-scale levels. A total of nine parameters are taken in account for SA of the mesoscale model and the change in collapse load is monitored. Milani et al. [11] conducted local SA for different types of masonry vaults. The parameters taken in consideration are tensile strength of mortar and coefficient of friction of mortar. Reddy et al. [12] studied the influence of elastic properties of the constituents and joint thickness on the strength of soil-cement block masonry. Milani and Benasciutti [13] investigated the application of Response Surface (RS) models in Monte Carlo analysis of complex masonry buildings with random input parameters. The accuracy of the estimated RS approximation, as well as the good estimations of the collapse load cumulative distributions are evaluated through polynomial RS models. Interestingly, in most of the above-mentioned literature, conclusions are drawn based on finite difference approximation, which is simplest form of local SA. In addition, no works relating to global SA of masonry are reported so far, and the application of High Dimensional Model Representation (HDMR) for conducting global SA to solve structural engineering problems has not been explored either.

In the current work, global SA is carried out on a mesoscale model of an unreinforced masonry (URM) shear wall using HDMR. Often mesoscale models contain large number of input parameters. This limits the application of the models to wider range of problems since knowledge regarding all the input parameters might not be available. Identifying the important parameters by global SA can resolve this issue. Moreover, decisions regarding physical structural systems can be made. This paper addresses these issues and proposes a novel strategy to evaluate the global sensitivity of the parameters with the least computational effort.

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In this paper, the method of Sobol' [14] is discussed in the second section, followed by a brief introduction to HDMR and its variations. Later, the procedure for calculation of sensitivity indices using HDMR is explored. In the fifth section, a mathematical example is solved using the method of Sobol'. In the sixth section, the finite element analysis of a mesoscale model of an URM shear wall is carried out and validated. Finally, global SA of an URM shear wall is carried out and conclusions are drawn.

2. Method of Sobol'

Sobol' [14] developed a general technique to conduct SA by introducing the concept of sensitivity indices. The method of Sobol' is a variance based method where the response $f(\boldsymbol{x})$ is expressed in terms of summands of increasing dimensionality in a unit hypercube $K^N = [0, 1]$ as follows,

$$f(\boldsymbol{x}) = f_0 + \sum_{i=1}^{N} f_i(x_i) + \sum_{1\le i\le j\le N} f_{ij}(x_i, x_j) + \cdots + f_{1,2,\ldots,N}(x_1, x_2 \ldots x_N). \tag{1}$$

Eq. (1) is known as ANOVA-decomposition (Analysis of Variance) or ANOVA-representation. All the terms in ANOVA-representation can be evaluated using multidimensional integrals as follows,

$$f_0 = \int_{K^N} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}, \tag{2}$$

$$f_i(x_i) = \int_0^1 \cdots \int_0^1 f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}^i - f_0, \tag{3}$$

$$f_{ij}(x_i, x_j) = \int_0^1 \cdots \int_0^1 f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}^{ij} - f_i(x_i) - f_j(x_j) - f_0, \tag{4}$$

and so on, where $\mathrm{d}\boldsymbol{x}$ is the product of $\mathrm{d}x_1, \mathrm{d}x_2, \mathrm{d}x_3, \ldots, \mathrm{d}x_N$, $\mathrm{d}\boldsymbol{x}^i$ stands for the product of $\mathrm{d}x_1, \mathrm{d}x_2, \mathrm{d}x_3, \ldots, \mathrm{d}x_N$ without $\mathrm{d}x_i$. Similarly, $\mathrm{d}\boldsymbol{x}^{ij}$ denotes the same product without $\mathrm{d}x_i$ and $\mathrm{d}x_j$. The total variance can be calculated as follows [14,15]

$$D = \int_{K^N} f^2(\boldsymbol{x})\mathrm{d}\boldsymbol{x} - f_0^2. \tag{5}$$

And the partial variances can be expressed as,

$$D_i = \int_0^1 f_i^2(x_i)\mathrm{d}x_i, \tag{6}$$

$$D_{ij} = \int_0^1 \int_0^1 f_{ij}^2(x_i, x_j)\mathrm{d}x_i\mathrm{d}x_j, \tag{7}$$

and so on. Squaring and integrating Eq. (1) over the whole domain K^N results in:

$$D = \sum_{i=1}^{N} D_i + \sum_{1\le i\le j\le N} D_{ij} + \cdots + D_{1,2,\ldots,N}. \tag{8}$$

The global sensitivity indices are defined as,

$$S_{i_1,\ldots,i_s} = \frac{D_{i_1,\ldots,i_s}}{D}. \tag{9}$$

The sensitivity indices are non-negative and their summation equals unity as follows,

$$\sum_{i=1}^{N} S_i + \sum_{1\le i\le j\le N} S_{ij} + \cdots + S_{1,2,\ldots,N} = 1. \tag{10}$$

3. High dimensional model representation

Understanding of any physical phenomenon involves identifying the input variables and output response as well as mapping the input–output (IO) relationship. As the number of input variable increases, the computational complexity in mapping IO relationship increases exponentially. This is commonly termed as *curse of dimensionality* [16–18]. HDMR [16–20] drastically reduces the computation effort by expressing the response in terms of hierarchical correlated function expansions.

HDMR expansion is expressed in terms of summation of component functions which are ordered starting from a constant term to gradually approaching to multivariance as the order of approximation is increased. Let $\boldsymbol{x} = [x_1, x_2, x_3, \ldots, x_N]$ be an N dimensional vector with N ranging up to 10^2–10^3. HDMR expresses the response function $f(\boldsymbol{x})$ as follows,

$$f(\boldsymbol{x}) = f_0 + \sum_{i=1}^{N} f_i(x_i) + \sum_{1\le i\le j\le N} f_{ij}(x_i, x_j) + \sum_{1\le i\le j\le k\le N} f_{ijk}(x_i, x_j, x_k) + \cdots + f_{1,2,\ldots,N}(x_1, x_2, \ldots, x_N), \tag{11}$$

where f_0 stands for the response at the mean of the input sample space. $f_i(x_i)$ is the first-order term expressing the response on varying x_i independently (although nonlinearly) over the sample space. Similarly, the function $f_{ij}(x_i, x_j)$ takes into account the correlation between x_i and x_j. The last term indicates residual contribution of Nth order correlated contribution of all the input variables. The basic conjecture underlying HDMR is that in real problems lower order correlations have more influence than higher order correlations [16,17].

HDMR expansions can be broadly categorized as RS (Random Sampling)-HDMR and Cut-HDMR. In the first case, the component functions are determined by integrating over randomly scattered sample points. Whereas, the component functions in Cut-HDMR expansion are evaluated along lines or planes or volumes (i.e. cuts) with respect to a reference point in sample space.

3.1. RS–HDMR

RS–HDMR expansion can be constructed when sample points are random in nature in a domain $\Re^N$. In this expansion, component functions are determined through averaging process. All the input variables are rescaled such a way that $0 \le x_i \le 1$. The response function is defined in the unit hypercube $K^N = \{(x_1, x_2 \cdots x_N) | 0 \le x_i \le 1,\ i = 1, 2, 3, \ldots, N\}$. The component functions are expressed as,

$$f_0 = \int_{K^N} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}, \tag{12}$$

$$f_i(x_i) = \int_{K^{N-1}} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}^i - f_0, \tag{13}$$

$$f_{ij}(x_i, x_j) = \int_{K^{N-2}} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}^{ij} - f_i(x_i) - f_j(x_j) - f_0, \tag{14}$$

and so on. The last term is calculated from the difference between $f(\boldsymbol{x})$ and all other component functions. f_0 can be approximated by finding the average of $f(\boldsymbol{x})$ for all value of $x^{(s)} = (x_1^{(s)}, x_2^{(s)}, x_3^{(s)}, \ldots, x_N^{(s)})$, $s = 1, 2, 3, \ldots, n$, where n is the number of sample points.

$$f_0 = \int_{K^N} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \approx \frac{1}{n}\sum_{s=1}^{n} f(\boldsymbol{x}^{(s)}). \tag{15}$$