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A sensitivity analysis method to compute the residual covariance matrix

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ABSTRACT

In state estimation, the covariance matrix of residuals is used to compute the normalized residuals and to detect erroneous measurements. This paper describes a method based on sensitivity analysis that allows computing the residual covariance matrix. The proposed method is estimator-independent, i.e., it is suitable for most solution approaches based on mathematical programming procedures. Several case studies illustrate the technique proposed. Relevant conclusions are finally drawn.

1. Introduction

1.1. Motivation and aim

State estimation consists in processing a given set of measurements to obtain the optimal estimate of the power system state. Several state estimation methods are proposed in the technical literature. Most of them are based on solving an optimization problem, such as the following methods: the Weighted Least Squares (WLS), the Least Absolute Value (LAV), the Least Median of Squares (LMS), the Least Trimmed of Squares (LTS), the Quadratic-Constant Criterion (QCC), and the Quadratic-Linear Criterion (QLC).

Measurements may contain gross errors due to various reasons. Thus, an essential feature of any state estimator is to detect those gross measurement errors, and, if possible, to identify and eliminate them. In general, bad measurement identification procedures rely on the residual covariance matrix and on the subsequent residual normalization. However, residual covariance matrix computation techniques differ across the methods. Moreover, these techniques usually compute an approximate residual covariance matrix using a first-order approximation and generally disregarding constraints. To overcome these drawbacks is the aim of this paper; i.e., to propose a novel estimator-independent procedure to compute accurately the residual covariance matrix.

1.2. Literature review and contribution

The technical literature is rich in references pertaining to state estimation techniques and algorithms [1–20]. Particularly notorious is the Weighted Least Squares estimator, which is a non-robust method exhaustively studied in the literature [1–12]. The objective function of the WLS estimator is computed as the sum of all squared measurement errors. This estimator is non-robust since a single outlier can distort significantly the estimation results.

Alternative approaches rely on the use of robust estimators, i.e., procedures less sensitive to bad measurements or outliers than the WLS technique. Some of them are based on minimizing a non-quadratic function of measurement residuals. The Quadratic-Constant and Quadratic-Linear Criterions belong to this category [13,14]. The Least Absolute Value method also belongs to this category and has gained widespread relevance thanks to its implicit bad data rejection property [15–17]. The objective function of the LAV estimator is computed as the sum of the absolute value of all measurement errors.

The Least Median of Squares [18–19] and the Least Trimmed of Squares [20] estimators are members of the same family. The objective function of the LMS estimator is computed as the squared measurement error whose value is the median of all measurement errors. The objective function of the LTS estimator is computed as the sum of the squared measurement errors whose values are lower than or equal to the median of all measurement errors. These methods are also capable of eliminating the effect of leverage points [11]. A measurement that highly influences the estimator outcome is denominated leverage point. In other words, a small perturbation

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in this measurement results in a significant change of the estimated state.

For any particular estimator, the technical literature provides estimator-specific techniques to compute the residual covariance matrix. For example, [21,22] and [15] report methods to compute this matrix for the unconstrained-WLS, constrained-WLS, and LAV estimators, respectively. For case of high-breakdown point estimators, [23] and [24] provide procedures to compute the asymptotic covariance residual matrices. To the best of our knowledge, no previous work proposes an estimator-independent method to compute this matrix.

The traditional techniques to compute the residual covariance matrix are specific for each estimator, and generally disregard the effect of second-order derivatives and constraints.

The specific contribution of this paper is to provide a method to compute the residual covariance matrix that (i) can be applied to any optimization-based state estimator, (ii) considers the influence of the second-order derivatives, and (iii) takes into account the equality/inequality constraints.

1.3. Paper organization

The rest of this paper is organized as follows. In Section 2, the state estimation problem is formulated as a general mathematical programming problem. In Section 3, the methodology for obtaining the residual covariance matrix and the normalized residuals is developed. Section 4 provides the expressions for calculating the derivatives of the estimates of the state variables with respect to the measurement values, which are needed for the estimation of the residual covariance matrix. In Section 5, the proposed technique is particularized for the well-known WLS and LAV methods. Section 6 provides results from three case studies to illustrate the performance of the proposed method. Finally, Section 7 provides some relevant conclusions.

2. State estimation formulation

Most state estimation models in practical use are stated as mathematical programming problems. These problems are formulated, in general, as:

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad J(\boldsymbol{y}) \tag{1}$$

subject to:

$$\boldsymbol{l}(\boldsymbol{x},\,\boldsymbol{z}) = 0 \tag{2}$$

$$\boldsymbol{g}(\boldsymbol{x},\,\boldsymbol{z}) \le 0 \tag{3}$$

where $\boldsymbol{z}$ is the $m \times 1$ measurement vector, $\boldsymbol{x}$ is the $n \times 1$ state-variable vector (variables to be estimated), $\boldsymbol{y}$ is the difference vector between the measurement and the functional vectors (a functional vector is a vector whose elements are functions), i.e., $\boldsymbol{y} = \boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})$, $J(\boldsymbol{y})$, is the objective function defined by the estimator, $\boldsymbol{l}(\boldsymbol{x},\,\boldsymbol{z})$ are equality constraints, e.g., to model zero-injection buses, and $\boldsymbol{g}(\boldsymbol{x},\,\boldsymbol{z})$ are inequality constraints, e.g., physical limits. Parameters p and q correspond to the number of equality and inequality constraints, respectively.

The solution of problem (1)–(3) provides the optimal estimate of the system state, $\hat{\boldsymbol{x}}$, which is assumed to be close enough to the true state $\boldsymbol{x}^{\text{true}}$. The residual vector $\boldsymbol{r}$ is defined as:

$$\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{h}(\hat{\boldsymbol{x}}) \tag{4}$$

Note that $\boldsymbol{r} = \boldsymbol{y}|_{\boldsymbol{x}=\hat{\boldsymbol{x}}}$.

Using the general model (1)–(3) this paper provides a procedure to compute the residual covariance matrix based on sensitivity analysis [25].

3. Residual covariance matrix and residual normalization

Using a first-order Taylor expansion of function $\boldsymbol{h}(\boldsymbol{x})$ around the optimal state vector $\hat{\boldsymbol{x}}$, the differential residual vector is obtained from (4) as:

$$d\boldsymbol{r} = dz - \left.\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\hat{\boldsymbol{x}}} d\hat{\boldsymbol{x}} = dz - \boldsymbol{H}d\hat{\boldsymbol{x}} \tag{5}$$

where $\boldsymbol{H}$ is the $m \times n$ Jacobian measurement matrix evaluated at $\hat{x}$.

From (5), it readily follows:

$$\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{z}} = \boldsymbol{I} - \boldsymbol{H}\left.\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{z}}\right|_{\boldsymbol{x}=\hat{\boldsymbol{x}}} = I - \boldsymbol{H}\boldsymbol{M}_{xz} = \boldsymbol{S} \tag{6}$$

where matrix $\boldsymbol{M}_{xz}$ is made of the derivatives of the state estimator vector $\boldsymbol{x}$ with respect to measurements $\boldsymbol{z}$ evaluated at $\hat{\boldsymbol{x}}$, matrix $\boldsymbol{I}$ is the m-dimensional identity matrix, and matrix $\boldsymbol{S}$ is known as the residual *sensitivity* matrix.

Note that (6) allows calculating matrix $\boldsymbol{S}$ for the general model (1)–(3).

The linear transformation from $\boldsymbol{z}$ to $\boldsymbol{r}$ at the optimum is obtained throughout the integration of (6):

$$\boldsymbol{r} = \boldsymbol{S}\boldsymbol{z} + \boldsymbol{k} \tag{7}$$

where $\boldsymbol{k}$ is the integration constant vector.

The expected value of the residual vector in (7) is:

$$\text{E}[\boldsymbol{r}] = \boldsymbol{S}\text{E}[\boldsymbol{z}] + \boldsymbol{k} \tag{8}$$

and, subtracting (8) from (7), it readily follows:

$$\boldsymbol{r} - \text{E}[\boldsymbol{r}] = \boldsymbol{S}(\boldsymbol{z} - \text{E}[\boldsymbol{z}]). \tag{9}$$

From (9), the residual covariance matrix $\boldsymbol{\Omega}$ is:

$$\begin{aligned} \boldsymbol{\Omega} &= \text{E}[(\boldsymbol{r} - \text{E}[\boldsymbol{r}])(\boldsymbol{r} - \text{E}[\boldsymbol{r}])^T] \\ &= \text{E}[(\boldsymbol{S}(\boldsymbol{z} - \text{E}[\boldsymbol{z}]))(\boldsymbol{S}(\boldsymbol{z} - \text{E}[\boldsymbol{z}]))^T] \\ &= \boldsymbol{S}\text{E}[(\boldsymbol{z} - \text{E}[\boldsymbol{z}])(\boldsymbol{z} - \text{E}[\boldsymbol{z}]^T)]\boldsymbol{S}^T \\ &= \boldsymbol{S}\boldsymbol{C}_z\boldsymbol{S}^T \end{aligned} \tag{10}$$

where matrix $\boldsymbol{C}_z$ is the measurement covariance matrix.

Therefore, considering (6), the general expression of matrix $\boldsymbol{\Omega}$ is:

$$\boldsymbol{\Omega} = (\boldsymbol{I} - \boldsymbol{H}\boldsymbol{M}_{xz})\boldsymbol{C}_z(\boldsymbol{I} - \boldsymbol{H}\boldsymbol{M}_{xz})^T. \tag{11}$$

Note that matrix $\boldsymbol{M}_{xz}$ depends on the estimator used, whereas matrix $\boldsymbol{H}$ is computed in the same way for all estimators. The main contribution of this paper is to provide an estimator-independent procedure to obtain matrix $\boldsymbol{M}_{xz}$.

Finally, from (4) and (11), normalized residuals are computed as

$$r_i^N = \frac{r_i}{\sqrt{[\boldsymbol{\Omega}]_{(i,i)}}} = \frac{z_i - h_i(x)}{\sqrt{[\boldsymbol{\Omega}]_{(i,i)}}} \quad i = 1, \ldots, m. \tag{12}$$

Vector r^N corresponds to the normalized residuals, and it can be used straightforwardly for bad data identification.

4. Derivatives of the state variables with respect to the measurements

As shown in Section 3, to compute the sensitivity matrix $\boldsymbol{S}$, which allows calculating the residual covariance matrix $\boldsymbol{\Omega}$, matrix $\boldsymbol{M}_{xz}$ is required. This matrix is obtained below, based on sensitivity analysis results reported in [26]. The technique to obtain matrix $\boldsymbol{M}_{xz}$ constitutes the main contribution of this paper.

The optimal primal/dual solution of problem (1)–(3) is denoted as $(\hat{\boldsymbol{x}},\,\boldsymbol{\lambda})$, where $\boldsymbol{\lambda}$ is the dual variable vector related to both equality