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A sequential algorithm and error sensitivity analysis for the inverse heat conduction problems with multiple heat sources

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ABSTRACT

This paper proposes a sequential approach to determine the unknown parameters for inverse heat conduction problems which have multiple time-dependent heat sources. There are two main aims in this study, one is to derive an inverse algorithm that can estimate the unknown conditions effectively, and the other is to bring up a theoretical sensitivity analysis to discuss what causes the growth of errors. This paper has three major achievements with regard to the literature on IHCPs, as follows: (1) proposing an efficient sequential inverse algorithm that can simultaneously determine several unknown time-dependent parameters; (2) exploring why the sequential function specification method can provide a stable but inaccurate estimation when tackling problems with larger measurement errors; and (3) discussing the sensitivity problem and analyzing what factors cause the growth in error sensitivity. Three examples are applied to demonstrate the performance of the proposed method, and the numerical results show that the accurate estimations can be obtained by alleviating the error sensitivity when the measurement error is considered.

1. Introduction

Inverse heat conduction problems (IHCPs) have been widely discussed in many scientific and practical engineering fields over the past few decades. For IHCPs studies, the main work is to deal with the determination of unknown parameters, such as the strength of the interior heat source, the boundary temperature, the boundary heat flux, the thermal property, the geometry boundary and so on, and the results of such works have many industrial applications. In the processes of finding unknown parameters, however, a crucial challenge is that a small input disturbance would lead to a large output error or undesirable dispersion. Namely, the typical ill-posed feature for inverse problems must be subdued in order to achieve an accurate solution. Therefore, dealing with the issues estimation stability and sensitivity analysis is essential in solving inverse heat conduction problems. Various approaches have been developed to stabilize the numerical inverse solution, and these have been widely applied to different kinds of inverse problems, such as Tikhonov's regularization [1,2], gradient iterative regularization [3,4], and the sequential function specification method (SFSM) [5–8]. In addition, grey prediction [9] and genetic algorithms [10] have also been employed to stabilize the numerical inverse solution.

This study includes two main aims, one is to derive an inverse algorithm that can estimate the unknown time-dependent conditions effectively, and the other is to bring up a theoretical sensitivity analysis to discuss what causes the growth of errors. The sequential function specification method is employed in this study due to its high computational efficiency, which is capable of stabilizing the fluctuation of ill-posed effects, and does not require an iterative procedure. The method

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Nomenclature

$\bar{k}$	thermal conductivity
$\bar{k}_r$	reference thermal conductivity
M	sensitivity matrix
P	defined as Eq. (19)
Q	heat source
r	number of future time
T	temperature
T_0	initial temperature
t	temporal coordinate
x, y	spatial coordinates
Y	measured temperature
Greek symbols	
φ	component of ψ
γ	component of matrix P
λ	random real number between 1 and -1
σ	standard deviation of measurement error
$\hat{\psi}$	estimated source strength
Δt	increment of temporal domain
$\Delta x, \Delta y$	increment of spatial domain
Ψ	unknown vector of heat source
Θ	vector
Subscripts	
i, j	coordinates of source
m	number of heat sources
p	number of measurement locations
Superscripts	
a, k, n, m	indices

has also been extensively applied in many studies for solving IHCPs. For example, Chantasiriwan [11] used the method to obtain the time-dependent Biot number from the solutions of determining boundary heat flux and boundary temperature. Yang [12] estimated the strength of the heat source for one-dimensional IHCPs based on symbolic computation, while Yang [13] applied the sequential method to determine two unknown time-dependent interior sources with two measurement sensors located on both boundary surfaces. One of the cases in Yang's study [13] was to find two unknown time-dependent heat sources with a high ratio of peak value, like 30 and 50, and a passable estimation was obtained. Yang [13] derived the discrete procedure in spatial coordinates with the finite element method (FEM) and the temporal coordinate with the finite difference method (FDM) to develop a mathematical algorithm, and then combined Beck's [1] concept of future time to stabilize the estimation. Lin and Yang [14] employed a modified Newton–Raphson method with the concept of future time to calculate the strength of the heat source for Fourier and non-Fourier heat conduction problems, and their results showed that more future times are needed in the hyperbolic equation than in the parabolic one. Although the estimations in those studies are fairly stable, only quite small measurement errors (for instance 0.001, 0.005 and 0.01) were considered, and their models were all only one-dimensional. Furthermore, they did not discuss the sensitivity problem, which has a significant influence on the determination of IHCPs.

This paper has three main achievements with regard to the literature on IHCPs, as follows:

- This study proposes an efficient sequential inverse algorithm that can simultaneously determine several unknown strengths of time-dependent heat sources and provide a stable and accurate estimation.
- This study also explores why the sequential function specification method would provide a stable, but inaccurate estimation when a large number of future times are used to tackle problems with larger measurement errors.
- This study discusses the sensitivity problem, and analyzes what factors cause the error sensitivity to grow. Meanwhile, the impacts of different measurement positions and various numbers of future times (1–15) are also demonstrated by examples to verify the theoretical analyzes.

This paper includes five sections. The present section makes brief statements and describes related research results in the literature on IHCPs, as well as outlining the three main achievements of this work. Section 2 presents the mathematical formulation to derive a sequential algorithm that is used to solve the problem of interest. Section 3 discusses the sensitivity