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# Sensitivity analysis of steel plane frames with initial imperfections

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ABSTRACT

The article presents the sensitivity and statistical analyses of the load-carrying capacity of a steel portal frame. It elaborates a typical stability problem of a system comprising two single-storey columns loaded in compression. The elements of this system mutually influence each other, and this fact, in conjunction with the random imperfections, influences the load-carrying capacity variance. This mutual interaction is analysed using the Sobol' sensitivity analysis. The Sobol' sensitivity analysis is applied to identify the dominant input random imperfections and their higher order interaction effects on the load-carrying capacity. Majority of imperfections were considered according to the results of experimental research. Realizations of initial imperfections were simulated applying the Latin Hypercube Sampling method. The geometrical nonlinear solution providing numerical result per run was employed. The frame was meshed using beam elements. The columns of the plane frame are considered with two variants of boundary conditions. The dependence between mean and design load-carrying capacities and column non-dimensional slenderness is analysed.



## 1. Introduction

Along with the progress of structural design theories and the technological advancement of steelworks production, more and more large-scale and high-rise steel bar structures are implemented in modern structures. The issue of stability of these structures becomes more apparent due to the utilization of more slender members.

The frame stability requires that all structural members and connections of the frame have adequate strength to resist the applied loads where static equilibrium is satisfied on the deformed geometry of the structure. In order to determine the load-carrying capacity of an actual structure, it is necessary to take into consideration initial imperfections and to consider the geometrically nonlinear analysis.

In general, all imperfections are of random character. The reliability of steel structures depends on the variance of input imperfections which influences the evaluation of limit states of building structures. The attainment of limit states is generally a random phenomenon, which is examined in the field of reliability using probabilistic theories and mathematical computation models.

One of the most important characteristics occurring in probabilistic methods of reliability assessment of steel structures is the variance of the load-carrying capacity which is primarily given by the quality of production. Basic indicators of production quality include the yield strength, tensile strength, ductility and geometrical characteristics of cross sections; see, e.g., [1,2]. Relatively sufficient statistical information is provided for the material and geometrical characteristics of mass produced hot-rolled members of steel structures in comparison to other building branches. Scarcely measurable imperfections of steel plane frames include the inevitable initial crookedness of bar members (bow imperfections) and out-of-plumb inclinations of the columns (sway imperfections) in the same frame [3,4]. Some measurements have been made in connection with testing programmes [5], but very little data is available.

The frames depicted in Figs. 1 and 2 represent a typical stability problem of a structural system consisting of more members. The frames are typical lean-on systems which are characterized by the structural members tied or linked together in such a way that buckling of the column would require adjacent members to buckle with the same lateral displacement. The imperfection interaction effects can have a significant influence on the overall performance of the frames. The steel frame depicted in Fig. 1 has rotation and translation fixed boundary conditions of both column ends. The steel plane frame in Fig. 2 is similar to that in Fig. 1 with the exception that there is no rotation restrain at the column ends. The rotation fixed and rotation free conditions represent the two limits of real anchorage in practice. Let us denote the frame in Fig. 1 as Frame 1 and the frame in Fig. 2 as Frame 2.

In the presented paper, the effects of input imperfections on the load-carrying capacities of Frames 1 and 2 are evaluated

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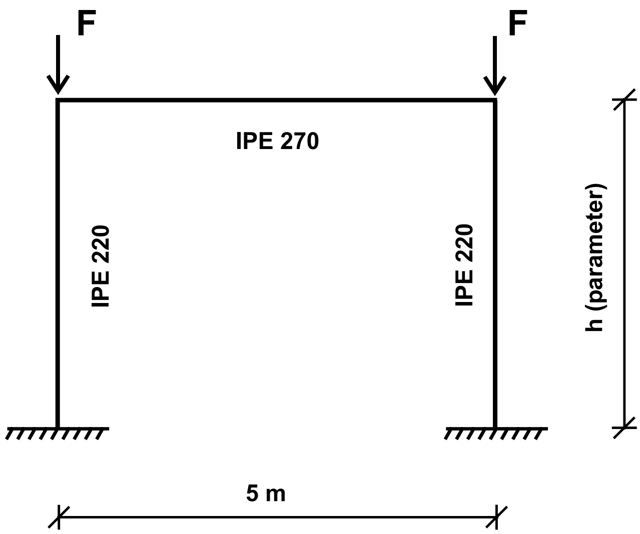


**Fig. 1.** Frame 1, rotation and translation fixed boundary conditions.

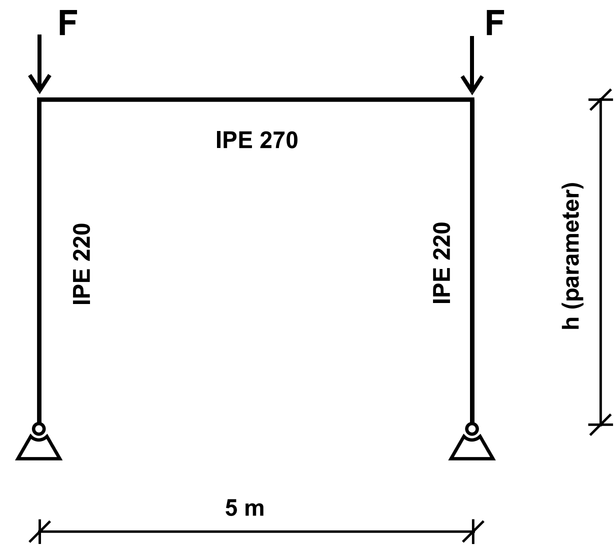


**Fig. 2.** Frame 2, rotation free and translation fixed boundary conditions.

by means of sensitivity analysis. The lean-on imperfect system (left column leans on right column) requires the utilization of sensitivity analysis which enables the evaluation of the influence of individual imperfections on the load-carrying capacity as well as of their higher order interaction effects. An outline of sensitivity analysis methods with examples of their application in a number of scientific fields is listed, e.g., in [6]. With regard to the random character of initial imperfections, the influence of imperfections on the load-carrying capacity of the frame systems will be studied applying the Sobol' sensitivity analysis [7–9]. One of the advantages of Sobol' sensitivity analysis is that it enables the identification of interaction effects among input quantities on the monitored output. The effects of the dominant imperfections, which have the greatest influence on the load-carrying capacities, will be described. Design load-carrying capacities evaluated statistically according to EN1990 [10] and according to the partial factor method of EUROCODE 3 [11] will be compared later on in the article. Obtained results will be discussed in connection with the results of Sobol' sensitivity analysis.

## 2. Input random imperfections

Imperfections are practically unavoidable, and they represent acceptable construction tolerances. The presence of imperfections in the analysis and design of frame systems with slender members has always been recognized, however, the manner of considering their effect on structural behaviour in computational models differs. Imperfections may generally be considered as deterministic (non-random) variables [12] or as random variables [4,13]. In this article, all imperfections will be considered as random variables.

Geometrical imperfections are, as a general rule, not visible to the naked eye, nor can they be quantified precisely beforehand. The first type of geometrical imperfection is the inclination of each column; see Fig. 3. Let us denote the inclination of the left column as $\Theta_1$ and of the right column as $\Theta_2$. Permitted inclination deviations of columns of a single-storey portal frame are listed in the standard EN1090-2. Let us assume that imperfections $\Theta_1$ and $\Theta_2$ are statistically independent random quantities with mean values equal to zero (perfectly vertical columns) and that both quantities have a Gauss probability density function. Let us further assume that 95% of realizations of $\Theta_1$ and $\Theta_2$ remain within the tolerance limits given by the standard EN1090-2. The classification Class 1 according to standard EN1090-2 was assumed. Detailed probabilistic derivation of the standard deviations $\sigma_{e_1}, \sigma_{e_2}$ is

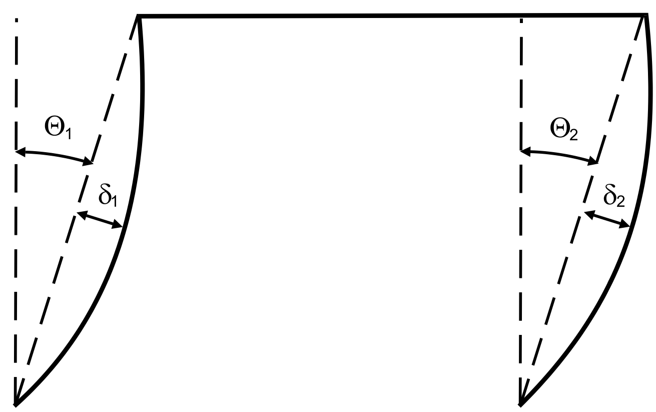


**Fig. 3.** Initial sway and bow imperfections.

described in [4]. Practical analysis based on the Monte Carlo simulation method may be performed in the following manner. Let us introduce quantities $\Theta_1$ and $\Theta_2$ with $\sigma_{e_1} = \sigma_{e_2} = h/790$. In the case that the random realizations of $\Theta_1$ and $\Theta_2$ have opposite signs, we shall multiply the inclination of each column by the coefficient 79/43; see [4].

The initial bow imperfection (initial crookedness) of member axis was described using a half sine wave; see Fig. 3. Both the positive and negative realizations of the amplitude should occur with the same frequency, which means that the mean value equals zero. The standard deviation of the Gauss probability density function has been selected for the random amplitude such that 95% of the realizations are found within the tolerance limits given by the standard EN 10034. Let us denote the amplitude of initial crookedness of left column as $\delta_1$ and the amplitude of right column as $\delta_2$.

Fig. 4 illustrates eight combinations of positive and negative imperfections from Fig. 3. Fig. 4 provides a basic idea of the shape but not of the magnitude of imperfect geometry. A more accurate notion would be obtained if imperfections $\Theta_1$, $\Theta_2$, $\delta_1$, $\delta_2$ were considered as random variables; see Fig. 5. Fig. 5 illustrates (in a larger scale) eight realizations of initial imperfections generated by the LHS method [14,15]. Random geometrical characteristics of profiles IPE were deduced according to results of experimental research [2]. According to standard EUROCODE 3, hot-rolled cross section is classified as Class 1 cross section which can form a plastic hinge with the rotation capacity required for plastic hinges. Effects