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Free vibration analysis and eigenvalues sensitivity analysis for the composite laminates with interfacial imperfection

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ABSTRACT

Free vibration analysis and eigenvalues sensitivity analysis of composite laminates with interfacial imperfection are investigated based on the radial point interpolation method (RPIM) in Hamilton system. The governing equation of the free vibration analysis and eigenvalues sensitivity analysis are both reduced by the spring-layer model and modified Hellinger–Reissner (H–R) variational principle. The analytical method (AM), semi-analytical method (SA) and the finite difference method (FD) are used for the eigenvalues sensitivity analysis in Hamilton system. Extensive numerical results are used to show the effects of variations in the material properties and shape parameters of the composite laminates on the response quantities and sensitivity coefficients of natural frequencies. A major advantage of the governing equation sets is that the interfacial imperfection of composite laminated plates is taken into account.

1. Introduction

Composite materials are used in almost all aspects of the industrial and commercial manufacturing fields of aircraft, ship, common vehicle and other high performance structures due to their high specific stiffness and strength, excellent fatigue resistance, longer durability as compared to metallic structures, and ability to be tailored for specific applications. The rapidly growing applications of composite materials have led to intensive study of the dynamic behavior and dynamics optimization under various conditions. Structural sensitivity concerns the relationship between design parameters and structural behaviors characterized by a response function. It is well known that the sensitivity analysis plays an important role in the general structural optimization. Using information obtained from design sensitivity analysis one can improve design greatly. Consequently, sensitivity analysis is currently one of the major research trends in computational mechanics.

Different plate theories have been developed for dynamics response analysis and sensitivity analysis of composite laminates. In these theories of dynamics for laminated thick plates, transverse shear deformation is very important such that some improved formulations which account for the deformation and rotator inertia have to be introduced in the analysis of dynamics. Since the above theories are established on some hypothesis, only partial fundamental equations can be satisfied and some of the elastic constants cannot be taken into account. Therefore, the errors will increase as the thickness of plate increases and the stress at interface cannot be exactly calculated.

In recent years, the state-vector equation in Hamilton system, which is employed in the analysis of control systems of current significance, has attracted the attention of a number of investigators who are interested in the problems of laminated structures [1–16]. In the state-vector equation, two types of variables (i.e., the transverse stresses and displacements) are synchronously considered in the control equation. And the thick plates/shells or the laminated plates/shells problems can be treated without any assumptions regarding displacements and stresses. The approach to interlaminar continuity is different from some of the Zig-Zag strategies. Due to the transfer matrix technique being employed, the solution provides an exact continuous transverse stresses and displacement field across the thickness of laminated structure. Another significant difference from the classical layer-wise methods is that the scale of the final governing equation system is independent of the thickness and the number of layers of a structure. Therefore, the state-vector equation is adopted for the free vibration analysis and eigenvalues sensitivity analysis of composite laminates with bonding imperfections in this paper.

In addition, interfacial imperfection is usually not taken into account in the traditional theory of structural analysis of composite laminates. However, multifarious interlaminar debondings like microcracks, inhomogeneities, and cavities may be introduced into the bond in the process of manufacture or service. During the service lifetime, these tiny flaws can get significant. To avoid the local failure of bond or the whole collapse of structure, therefore, the effect of imperfect interfaces on the structural behavior should be

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accurately evaluated. Some researchers presented the theory work on this problem, but it was only focused on the analytical methods and traditional numerical methods [17–19]. In recent years, Chen [20–23] has used the analytical methods and numerical methods to research the problem of interfacial imperfection for composite laminated plates in Hamilton system.

The objective of the present paper is to research the problem of the free vibration analysis and eigenvalues sensitivity analysis of composite laminates with interfacial imperfections based on the meshless method, the state-vector equation and the spring-layer model. Furthermore, the analytical method (AM), semi-analytical method (SA) and the finite difference method (FD) are developed for the eigenvalues sensitivity analysis in Hamilton system.

2. Meshless formulistic of Hamilton canonical equation and the spring-layers

2.1. Interpolation using radial basis functions

Consider a continuous function $u(\boldsymbol{x})$ defined on a 2D domain Ω has a set of suitably located nodes in it. An interpolation of $u(\boldsymbol{x})$ in the neighborhood of a point $\boldsymbol{X}_{\mathrm{Q}}$ using RBFs and polynomial basis is written as [24]:

$$u(\boldsymbol{x}) = \sum_{i=1}^{n} R_i(\boldsymbol{x})a_i + \sum_{j=1}^{m} p_j(\boldsymbol{x})b_j = \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{a} + \boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{b} \tag{1}$$

with the constraint

$$\sum_{i=1}^{n} p_j(\boldsymbol{x})a_i = \boldsymbol{p}_m^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{a} = \boldsymbol{0}, \quad j = 1, 2, \cdots, m \tag{2}$$

here, $R_i(\boldsymbol{x})$ is a radial basis function associated with node i (i.e., for the modified multiquadrics (MQ) used in this present work, $R(\boldsymbol{x}) = [r_i^2 + (\alpha_c d_c)^2]^q$, where α_c and q are shape parameters, and d_c is a characteristic length that relates to the nodal spacing in the local support domain of the point of interest $\boldsymbol{x}$, and it is usually the average nodal spacing for all the nodes in the local support domain); n is the number of nodes in the neighborhood of $\boldsymbol{X}_{\mathrm{Q}}$, $p_j(\boldsymbol{x})$ is a monomial in the space coordinates $\boldsymbol{x}^{\mathrm{T}} = [x, y]$; m is the number of monomial basis functions (usually $m < n$); $a_i(\boldsymbol{x}_{\mathrm{Q}})$ and $b_j(\boldsymbol{x}_{\mathrm{Q}})$, which varies with the point $\boldsymbol{X}_{\mathrm{Q}}$, are coefficients for $R_i(\boldsymbol{x})$ and $p_j(\boldsymbol{x})$, respectively.

In utilizing radial basis functions, several shape parameters need to be determined for good performance. In general, these parameters can be determined by numerical examinations for given types of problems. For example, Wang and Liu left the parameter q open to any real variable, and found that $q = 0.98$ or 1.03 led to good results in the analysis of two-dimensional solid and fluid mechanics problems in the Lagrangian system [25,26]. In the present work, the optimum values of the shape parameters are obtained by repetitious numerical experimentation for the present three-dimension model. We adopt the optimum values of shape parameters for the MQ determined as $\alpha_c = 0.03$ and $q = 1.03$.

Requiring that the function $u(\boldsymbol{x})$ given by Eq. (1) equals its value at n nodes in the vicinity of the point $\boldsymbol{X}_{\mathrm{Q}}$, we get a set of simultaneous linear algebraic equations for the coefficients $a_i(\boldsymbol{x}_{\mathrm{Q}})$ and $b_j(\boldsymbol{x}_{\mathrm{Q}})$.

If the constraint (2) is satisfied at the same time, a linear equation group including $n + m$ equations can be obtained for the coefficients $a_i(\boldsymbol{x}_{\mathrm{Q}})$ and $b_j(\boldsymbol{x}_{\mathrm{Q}})$. With the solution of this linear equation group, Eq. (1) can be written as follows:

$$u(\boldsymbol{x}) = \Phi^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{U}_s = \sum_{i=1}^{n} \phi_i u_i \tag{3}$$

where $\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x}) = \{\varphi_1(\boldsymbol{x}), \varphi_2(\boldsymbol{x}), \cdots, \varphi_n(\boldsymbol{x})\}$ is the RPIM shape function with respect to displacement volume of the field node [24]. $\boldsymbol{U}_s = [u_1\ u_2 \cdots u_n]$ is the values at the field node.

2.2. Meshless formulistic of Hamilton canonical equation

For isotropic, orthotropic or anisotropic elasticity solids, the modified H–R variational principle in three-dimensional Cartesian coordinate system can be expressed as [20–22]:

$$\delta\Pi = \delta\iiint_V (-\mathrm{P}^{\mathrm{T}} - \mathrm{Q}_{,z} - H)dV + \delta\int\int_{S_A} \oiint(\lambda_1^{\mathrm{T}}\boldsymbol{B}_{p\bar{q}} - \lambda_0^{\mathrm{T}}\boldsymbol{B}_{\bar{p}q})dS \tag{4}$$

where $\boldsymbol{Q} = [u\ v\ w]^{\mathrm{T}}$, uvw are the total displacement components along (x, y, z) coordinates, respectively. $\boldsymbol{P} = [\sigma_{xz}\ \sigma_{yz}\ \sigma_{zz}]^{\mathrm{T}}$, σ_{xz}, σ_{yz} and σ_{zz} are the out-of-plane (i.e., transverse) stresses. The superscript T signifies a matrix transposition. H is Hamiltonian. V is referred to the volume considered. S_A is the surface over the volume. $\boldsymbol{B}_{p\bar{q}} = [\,p_x(u-\bar{u})\quad p_y(v-\bar{v})\quad p_z(w-\bar{w})\,]^{\mathrm{T}}$, $p_i(i = x, y, z)$ are the stress boundary conditions in three coordinate directions, respectively. $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the prescribed displacement boundary conditions along $(x,\ y,\ z)$ coordinates, respectively. $\boldsymbol{B}_{\bar{p}q} = [\bar{p}_x u\ \bar{p}_y v\ \bar{p}_z w]^{\mathrm{T}}$, $\bar{p}_i(i = x, y, z)$ are the prescribed stress boundary conditions in the $(x,\ y,\ z)$ directions, respectively. $\lambda_1 = [\lambda_x - 1\ \lambda_y - 1\ \lambda_z - 1]^{\mathrm{T}}$ and $\lambda_0 = [\lambda_x\ \lambda_y\ \lambda_z]^{\mathrm{T}}$ are the characteristic coefficients which are introduced especially, in which the values of $\lambda_i(i = x, y, z)$ are 1 or 0. $\lambda_i(i = x, y, z) = 1$ refers to the stress boundary cases in three coordinates directions, respectively. $\lambda_i(i = x, y, z) = 0$ refers to the displacement boundary cases in three coordinates directions, respectively.

By using RPIM shape function, $\boldsymbol{P}$ and $\boldsymbol{Q}$ at any point can be written as follows:

$$\begin{Bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{N} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N} \end{bmatrix} \begin{Bmatrix} \boldsymbol{P}_{\mathrm{e}} \\ \boldsymbol{Q}_{\mathrm{e}} \end{Bmatrix} \tag{5}$$

where $\boldsymbol{N} = diag[\boldsymbol{\Phi}]_{3\times3}$; $\boldsymbol{P}_{\mathrm{e}} = [\sigma_{xz}^{\mathrm{e}}(z)\ \sigma_{yz}^{\mathrm{e}}(z)\ \sigma_{zz}^{\mathrm{e}}(z)]^{\mathrm{T}}$; $\boldsymbol{Q}_e = [\boldsymbol{u}^{\mathrm{e}}(z)\ \boldsymbol{v}^{\mathrm{e}}(z)\ \boldsymbol{w}^{\mathrm{e}}(z)]^{\mathrm{T}}$.Substitution for $\boldsymbol{P}$ and $\boldsymbol{Q}$ from Eq. (5) into Eq. (4), applying the tools of variational calculus and integrating, an equation set including two equations can be reduced from the first term of Eq. (4). Applying the tools of variational calculus and integrating by parts, the meshless formulistic of Hamilton canonical equation can be obtained from this equation set as follows:

$$\begin{bmatrix} \boldsymbol{C} & 0 \\ 0 & \boldsymbol{C} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}z} \begin{Bmatrix} \boldsymbol{P}_e(z) \\ \boldsymbol{Q}_e(z) \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_{11}^{\mathrm{T}} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & -\boldsymbol{A}_{11} \end{bmatrix} \begin{Bmatrix} \boldsymbol{P}_e(z) \\ \boldsymbol{Q}_e(z) \end{Bmatrix} + \begin{Bmatrix} \Xi_e \\ \boldsymbol{0} \end{Bmatrix} \tag{6}$$

where $\boldsymbol{C} = \boldsymbol{C}^{\mathrm{T}} = \int_\Omega \boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}\,dxdy$, $\boldsymbol{A}_{11}^{\mathrm{T}} = \int_\Omega (\boldsymbol{G}_1\boldsymbol{N})^{\mathrm{T}}\boldsymbol{N} + (\boldsymbol{G}_2\boldsymbol{N})^{\mathrm{T}}\chi_{21}\boldsymbol{N}dxdy$, $\boldsymbol{A}_{12} = \int_\Omega (\boldsymbol{G}_2\boldsymbol{N})^{\mathrm{T}}\chi_{22}^{\mathrm{T}}(\boldsymbol{G}_2\boldsymbol{N}) - \mathbf{N}^{\mathrm{T}}\Omega\mathbf{N}dxdy$, $\boldsymbol{A}_{21} = \int_\Omega \boldsymbol{N}^{\mathrm{T}}\boldsymbol{\chi}_{11}\boldsymbol{N}\,dxdy$, $\boldsymbol{\Xi}_e = \int_\Omega \boldsymbol{N}^{\mathrm{T}}\boldsymbol{F}\,dxdy$. $\boldsymbol{F} = -[f_x\ f_y\ f_z]^{\mathrm{T}}$ denotes the vector of the body forces. For the isotropic and orthotropic materials, the matrices $\boldsymbol{G}_1$, $\boldsymbol{G}_2$, $\boldsymbol{\chi}_{11}$, $\boldsymbol{\chi}_{21}$ and $\boldsymbol{\chi}_{22}$ are given in Eqs. (A.1) and (A.2).

From Eqs. (6) and (4), the boundary term can be reduced to the following form:

$$\delta\int\int_{S_A} (\lambda_1^{\mathrm{T}}\mathbf{B}_{p\bar{q}} - \lambda_0^{\mathrm{T}}\mathbf{B}_{\bar{p}q})dS = \delta\int\int_{S_A} \Big(\Big(((\mathbf{G}_3\mathbf{P})^{\mathrm{T}} + (\mathbf{G}_4\mathbf{Q})^{\mathrm{T}}\Big)(\mathbf{Q} - \bar{\mathbf{Q}}) - (\varLambda_0\bar{\mathbf{p}})^{\mathrm{T}}\mathbf{Q}\Big)dS \tag{7}$$

Here, $\overline{\boldsymbol{P}}_{\mathrm{e}} = [\bar{\sigma}_{xz}^{\mathrm{e}}(z)\ \bar{\sigma}_{yz}^{\mathrm{e}}(z)\ \bar{\sigma}_{zz}^{\mathrm{e}}(z)]^{\mathrm{T}}$; $\overline{\boldsymbol{Q}}_e = [\bar{\boldsymbol{u}}^{\mathrm{e}}(z)\bar{\boldsymbol{v}}^{\mathrm{e}}(z)\bar{\boldsymbol{w}}^{\mathrm{e}}(z)]^{\mathrm{T}}$; $\Lambda_0 = \mathrm{diag}[\lambda_x\lambda_y\lambda_z]$; The explicit forms of the matrices $\boldsymbol{G}_3$, $\boldsymbol{G}_4$ and $\boldsymbol{\varLambda}_0$ are given in Eq. (A.3).

Substituting Eq. (5) into Eq. (7) and then carrying out variations, the matrix form of boundary term is:

$$\begin{bmatrix} \boldsymbol{B}_{11}^{\mathrm{T}} & \boldsymbol{B}_{12} \\ \boldsymbol{0} & -\boldsymbol{B}_{11} \end{bmatrix} \begin{Bmatrix} \mathbf{P}_{\mathrm{e}}(z) \\ \mathbf{Q}_{\mathrm{e}}(z) \end{Bmatrix} + \oint_{S_A} \begin{bmatrix} -\boldsymbol{N}^{\mathrm{T}}\Lambda_0 & -(\boldsymbol{G}_4\boldsymbol{N})^{\mathrm{T}} \\ \boldsymbol{0} & (\boldsymbol{G}_3\boldsymbol{N})^{\mathrm{T}} \end{bmatrix} \begin{Bmatrix} \overline{\boldsymbol{P}}_{\mathrm{e}} \\ \overline{\boldsymbol{Q}}_{\mathrm{e}} \end{Bmatrix} dS \tag{8}$$

where $\boldsymbol{B}_{11}^{\mathrm{T}} = \oint_{S_A}(\boldsymbol{G}_3\boldsymbol{N})^{\mathrm{T}}\boldsymbol{N}dS$; $\mathbf{B}_{12} = \oint_{S_A}\boldsymbol{N}^{\mathrm{T}}(\boldsymbol{G}_4 + \boldsymbol{G}_4^{\mathrm{T}})\boldsymbol{N}dS$.

Add Eq. (8) to the right hand side of Eq. (6), a total meshless formulistic of Hamilton canonical equation can be reduced. For the