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Vibration sensitivity analysis of MEMS vibratory ring gyroscopes

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ABSTRACT

This paper presents a detailed model for possible vibration effects on MEMS degenerate gyroscopes represented by vibratory ring gyroscopes. Ring gyroscopes are believed to be relatively vibration-insensitive because the vibration modes utilized during gyro operation are decoupled from the modes excited by environmental vibration. Our model incorporates four vibration modes needed to describe vibration-induced errors: two flexural modes (for gyro operation) and two translation modes (excited by external vibration). The four-mode dynamical model for ring gyroscopes is derived using Lagrange's equations. The model considers all elements comprising a ring gyroscope, namely the ring structure, the support-spring structures, and the electrodes that surround the ring structure. Inspection of this model demonstrates that the output of a ring gyroscope is fundamentally insensitive to vibration due to the decoupled dynamics governing ring translation versus ring flexure, however, becomes vibration-sensitive in the presence of non-proportional damping and/or capacitive nonlinearity at the sense electrodes.

1. Introduction

Mechanical vibrations potentially degrade the performance of microelectromechanical systems (MEMS) devices because the performance frequently relies on the displacements of or stress in microstructures. The undesirable dynamics induced by external vibration generates errors in the device output. Such vibration-induced output errors, also called vibration sensitivity, have been reported for various MEMS sensors and actuators. MEMS vibratory gyroscopes, because of their high quality factor (Q-factor), are known to be susceptible to vibration. The high Q is beneficial in improving gyro performance but also amplifies vibration amplitudes at certain frequencies and increases output signal distortions. Whereas, degenerate MEMS gyroscopes are conceptually known to be less sensitive to environmental excitation because of inherently symmetric gyro structures [1,2].

Degenerate MEMS gyroscopes utilize a degenerate vibration mode pair as the drive and sense modes to maximize the energy transfer between the two modes. A degenerate pair of vibration modes refers to two modes that have distinct mode shapes but identical natural frequencies [3,4]. Many degenerate gyroscopes utilize the so-called wine-glass modes of a vibrating shell structure representing two flexural modes (i.e., drive and sense modes). Wine-glass modes arise in several shell-like structures, including common ring gyroscope designs [1,2,5]. The natural frequencies of axisymmetric ring structures arise as degenerate pairs when the structures are fabricated from isotropic materials [4,6].

Fig. 1 illustrates a conceptual view of a ring gyroscope. A vibratory ring gyroscope consists of a ring structure, support-spring structures, and electrodes surrounding the ring structure. The electrodes are used for drive, sense, or control of the gyro. The operation of the ring gyroscope relies on two elliptically shaped vibration modes, named the primary and secondary flexural modes, which are also called the drive and sense modes, respectively. The two flexural modes have identical natural frequencies due to the (assumed) symmetry of the ring. Several variations of this design are also reported in [5,7–11], but the basic concept remains the same.

Ring gyroscopes are known to have a low vibration sensitivity because external vibration excitation couples only weakly (if at all) to the two flexural modes [1,12,13]. This knowledge is based on the vibration modes of axisymmetric ring structures [14,15]. Nonetheless, several studies offer qualitative explanations of potential vibration-induced error sources in degenerate gyroscopes including hemispherical resonator gyros [16] and ring gyros [1]. Therefore, it is still crucial to analyze and understand the operation of ideal ring gyros subjected to external vibration.

Several studies report rigorous analyses of ring gyro operation [1,2,17,18] or the ring gyro's response to external vibration [1,19–21]. However, these prior studies only present models that consider only partial components compromising a ring gyroscope or do not include sufficient number of vibration modes needed to analyze both gyro operation and vibration-induced errors. For

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Nomenclature

$\vec{u}$	vibration-induced displacement. Also represents the total displacement of a point on the ring structure due to translation and flexure, $\vec{u} = \vec{u}^T + \vec{u}^F$
$\vec{u}^T$	displacement of the center of the ring structure due to translation
$\vec{u}^F$	displacement of a point on the ring structure due to flexure
u_x, u_y	scalar components of total displacement in Cartesian coordinates
u_r, u_θ	scalar components of total displacement in polar coordinates
u_1 and u_2, q_1 and q_2, Φ_1 and Φ_2	vibration-induced displacement, generalized (modal) coordinate, mode shape of 1st and 2nd translation modes
u_3 and u_4, q_3 and q_4, Φ_3 and Φ_4	vibration-induced displacement, generalized (modal) coordinate, mode shape of 1st and 2nd flexural modes
$\Phi_{r1}/\Phi_{\theta1}$, $\Phi_{r2}/\Phi_{\theta2}$, $\Phi_{r3}/\Phi_{\theta3}$, $\Phi_{r4}/\Phi_{\theta4}$	radial/tangential components of mode shapes of 1st and 2nd translation modes and 1st and 2nd flexural modes
Φ_{x1}/Φ_{y1}, Φ_{x2}/Φ_{y2}, Φ_{x3}/Φ_{y3}, Φ_{x4}/Φ_{y4}	X and Y components of mode shapes of 1st and 2nd translation modes and 1st and 2nd flexural modes
$X_0Y_0Z_0$	inertial frame of reference
XYZ	translating and rotating (non-inertial) frame of reference. The origin of this frame is the original center of the ring structure (prior to any translation or this structure)
$\vec{U}_{arb}$	position of a point on the ring structure (shown as U in Fig. 4) relative to the inertial frame $X_0Y_0Z_0$
$\vec{r}_0$	position vector from the origin of the inertial frame to the origin of the non-inertial frame (XYZ)
$\vec{r}_p$	position vector from the center of the ring structure to a point on the undeformed ring
R_{ring}, W_{ring}, and h_{ring}	radius, width and thickness of ring structure
r_{spring}	radius of support springs
θ_n and $\Delta\theta_n$	location of each electrode and the arc of the electrode in Fig. 7
g_0	equilibrium gap spacing of electrodes of a ring gyroscopes
g_d	equilibrium gap spacing of drive electrodes of a ring gyroscopes. Generally, $g_d = g_0$.
g_s	equilibrium gap spacing of sense electrodes of a ring gyroscopes. Generally, $g_s = g_0$.
C_n	capacitance of each electrode in a ring gyroscope
c_{ij},	modal damping coefficients (i, j = 1,2,3,4)
ε	permittivity

instance, models exist that consider only the mechanical ring structures and ignore the support-spring structures [17–20], consider mechanical structures without electrodes [20,21], or do not account for the vibration modes directly excited by external vibration [1,2,10]. The potential energy of the support-spring structure needs to be included because the flexural stiffness of each support spring is not negligible compare to the stiffness of a ring structure [2,21]. The electrostatic force from the electrodes is important in evaluating gyro performance and vibration sensitivity [10,22]. The gyro read-out circuits often utilize parallel-plate sensing mechanism that contributes a nonlinear behavior between sense capacitance and sense axis displacement. This capacitive nonlinearity generates vibration-induced errors in other types of gyroscopes, such as tuning fork gyroscopes [22]. Thus, it is essential to consider all components of ring gyroscopes to achieve a comprehensive understanding of vibration sensitivity of ring gyroscopes.

This paper fills this void by contributing a detailed model that includes all components of ring gyroscopes (including the ring structure, the support-beam structure, the electrodes, and damping). The model describe both gyro operation and its response to external vibration by employing Lagrange's equation and four vibration modes either representing ring gyro operation (named flexural modes) or excited by external linear vibration (named translation modes). This work analyzes ideally fabricated ring gyroscopes and represents a step toward a complete model to understand the vibration sensitivity of MEMS ring gyroscopes.

We open with an overview of this model in Section 2 and derive equations governing energies from or work done by each ring-gyro components described from Sections 2–6. The derived equations are analyzed using Lagrange's equation in Section 7. Furthermore, the effect of capacitive nonlinearity at sense electrodes is analyzed in Section 8. In addition, we also briefly discuss the effect of non-ideality including non-proportional damping (Section 6.2).

2. Overview of model

2.1. Ring gyroscope operation

The ring gyroscope, shown in Fig. 1, operates as follows. First the electrostatic drive is used to excite the primary flexural (drive) mode in resonance. When the device is subjected to superimposed rotation, a portion of the vibration energy is transferred from the primary flexural mode to the secondary flexural (sense) mode. The amplitude of the radial displacement of the secondary flexural mode is proportional to the rotation rate and thus this displacement serves as the means to detect the rotation rate. Environmental vibration may also excite other vibration modes, particularly the two ring translation modes which induce a rigid translation of the ring on its flexible support. Thus, the overall motion of the ring is decomposed into that excited by the gyro's operation (flexural modes) from that excited by environmental vibration (ring translation modes).

The support-spring structure in this illustrated design utilizes eight semicircular springs that attach the ring structure to the substrate at the center of the ring. The eight-spring design plays an important role in suspending the ring structure, in assuring balanced and symmetric operation of the ring gyroscope, and in allowing the two flexural modes to have identical natural frequencies. The necessity for using eight springs is discussed in previous works [1,2].

Unlike the non-degenerate gyroscopes, like tuning fork gyros, ring gyroscopes cannot be analyzed using simple lumped models because the mass and the stiffness of the ring gyro are distributed along the ring. Herein, we represent the ring as a continuous (curved) beam and represent each support-beam as a discrete spring attached to the ring mass.

2.2. Modeling approaches and assumptions

We model two primary classes of vibration modes: translation modes and flexural modes. Ring gyroscope designs (made of either Nickel [1] or polysilicon [2]) possess translation modes having resonant frequencies (∼20 kHz) much smaller than those of flexural modes (∼30 kHz). Therefore, the translation modes are more susceptible to external/environmental vibration whose frequency spectrum also lies well below these resonant frequencies. Of course, ultra-high frequency external vibration may also directly excite the flexural modes but this would indeed be rare, except